T-tests

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Testing

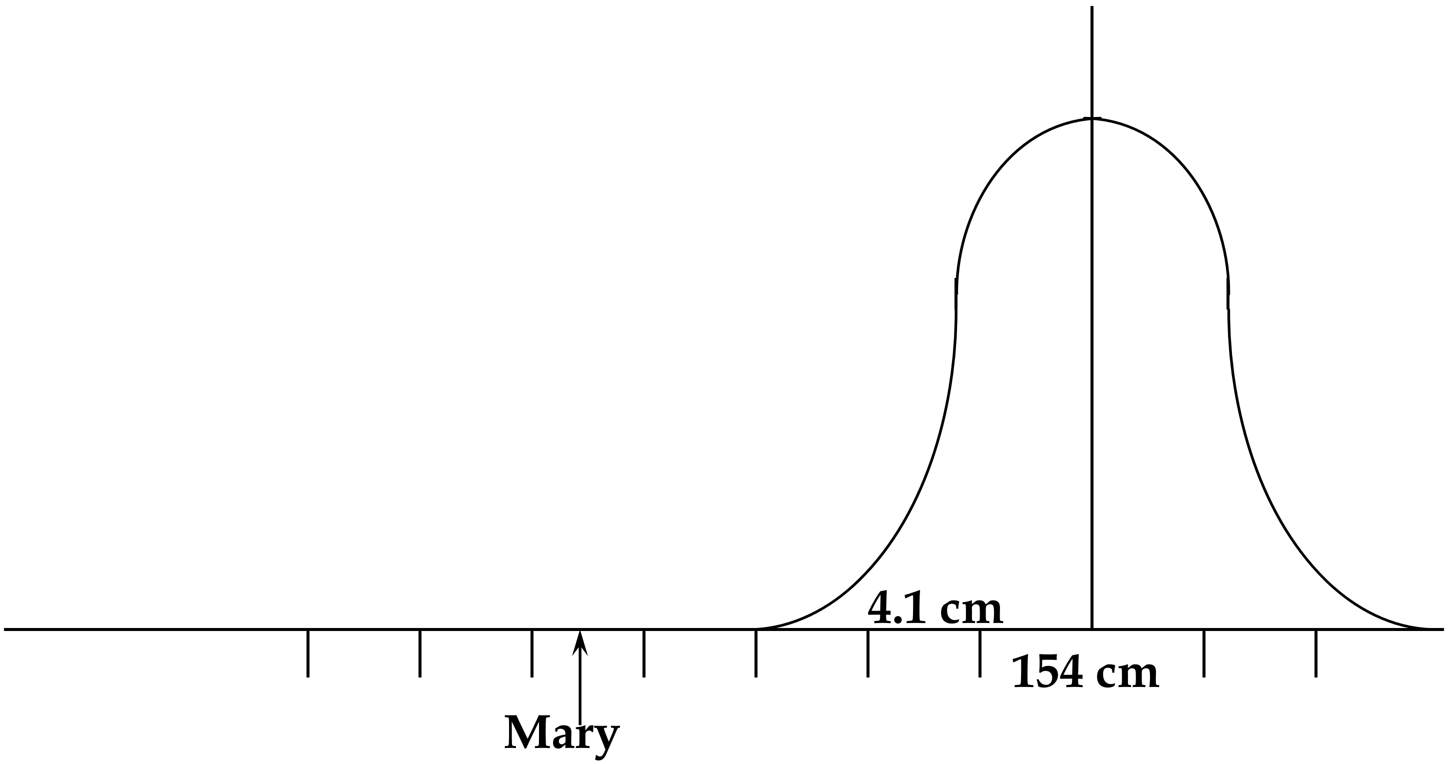
- We are no longer interested in estimating what the population mean actually could be, but…..
- Is the single value or group mean we have measured compatible with a specific population mean chosen by us??

Formula:

$$\frac{135\ \text{cm} - 154\ \text{cm}}{4.1\ \text{cm}} = -4{,}63$$

- Mary is almost five times further away from the chosen mean than random variation

Testing



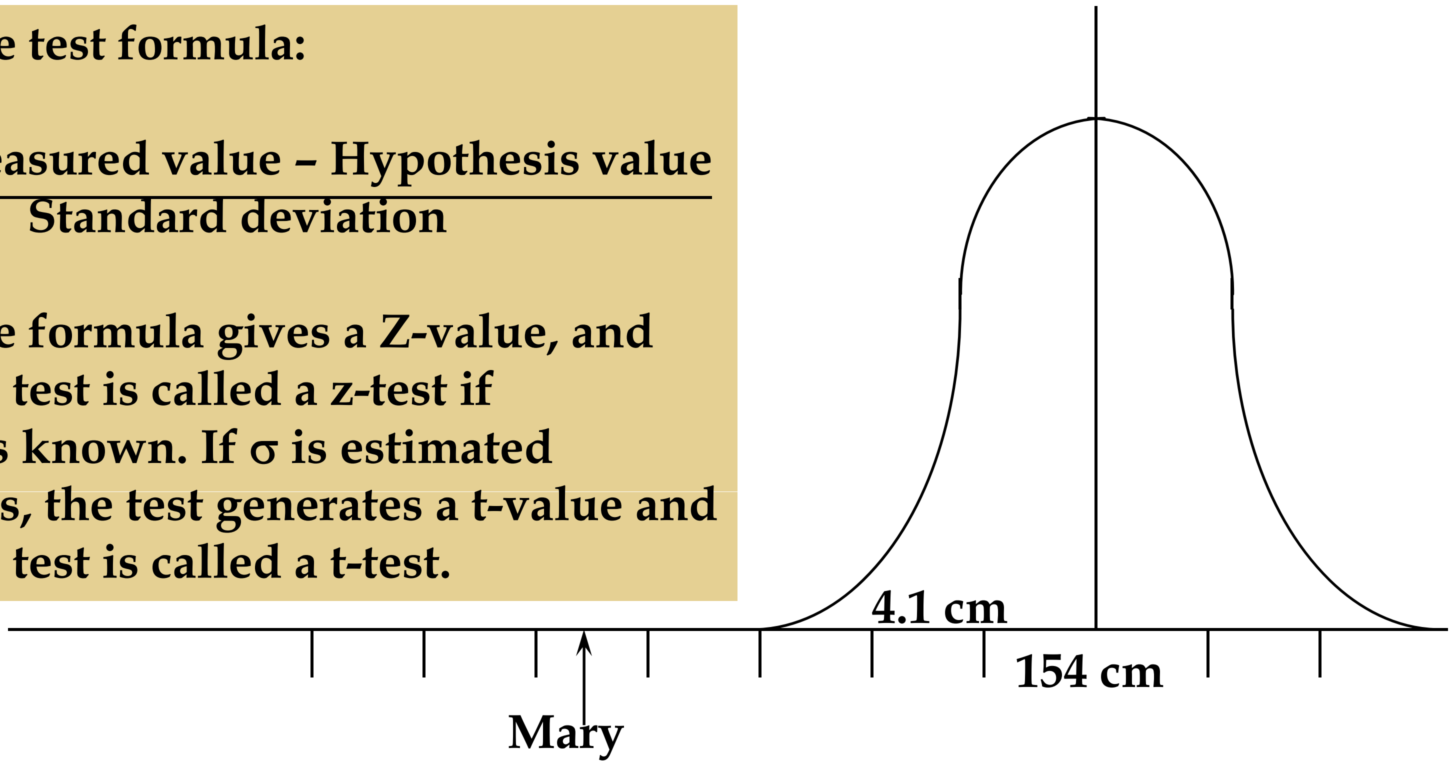
The probability of by chance seeing a girl with a height of 135 cm, given the population mean of 154 cm, is less than 0.0001

Testing

The test formula:

$$\frac{\text{Measured value} - \text{Hypothesis value}}{\text{Standard deviation}}$$

The formula gives a Z-value, and the test is called a z-test if σ is known. If σ is estimated by s, the test generates a t-value and the test is called a t-test.



Testing

- If the data are normally distributed, a probability can be found to correspond with this or any larger (or smaller) z-value
- This probability is given as a p-value (p stands for probability)
- In Mary’s case this p-value is smaller than 0.0001 indicating that less than one in 10000 girls will by chance be this short or even shorter if the population mean is 154cms.

Testing

- The same formula and principle applies if what we measure is a group mean.

$$\frac{\text{Group mean – Hypothesis value}}{\text{Standard deviation for group means (Standard error of the mean)}}$$

This is called a one-sample z-test if σ is known. If σ is estimated by s, The test is called a t-test.

Testing

- Let us say we have another class of 16 girls with heights: 148, 165, 157, 158, 160, 158, 153, 156, 157, 162, 167, 153, 159, 155, 159, 149 cms
- Their mean is 157.3 cms and SD 5.09
- Could they be a random sample from a population with mean 154 cms?

Testing

- Here is the formula:

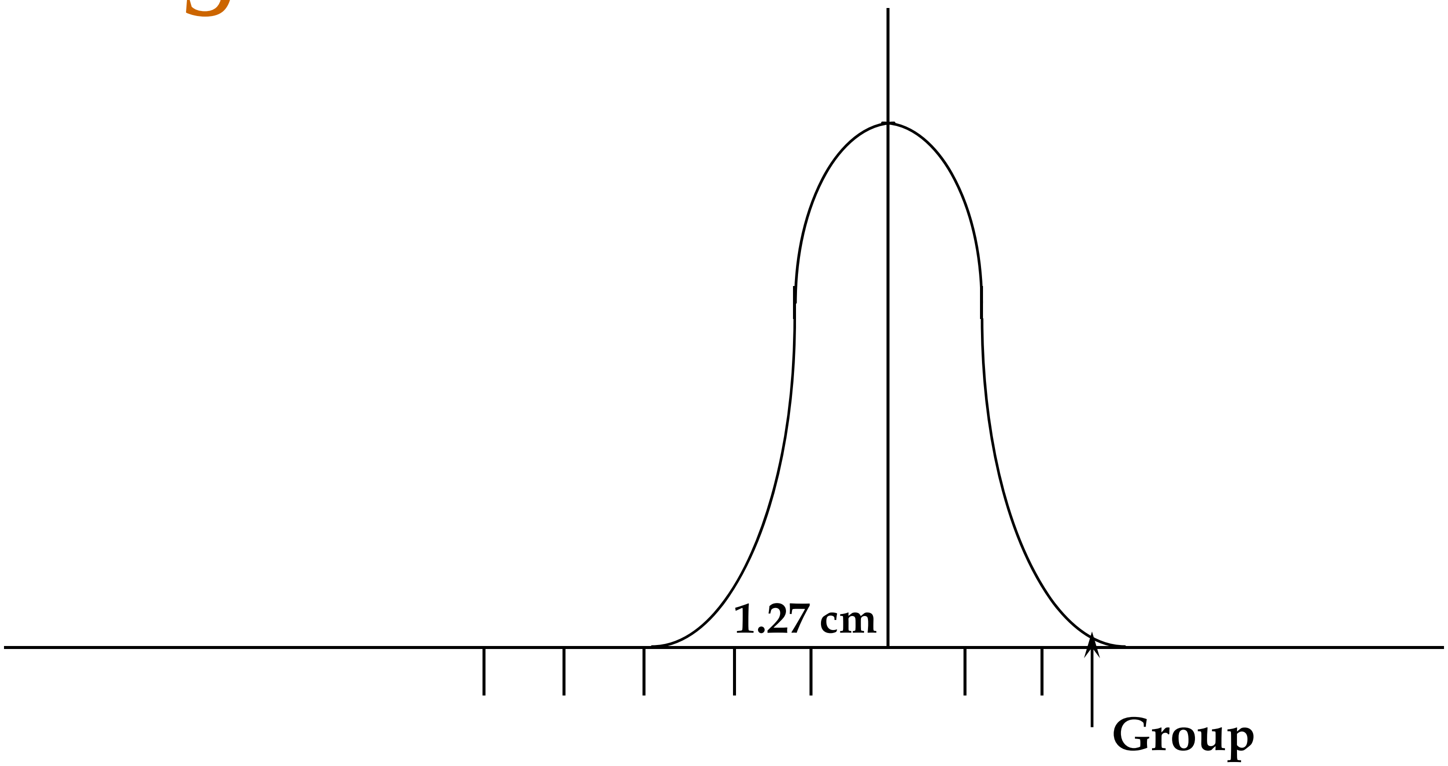
Measured value

Hypothesis value

$$t = \frac{157.3 - 154}{\frac{5.09}{\sqrt{16}}} = 2.59$$

Standard deviation

Testing



- The p-value is in this case 0.02

Testing

- A one-sample t-test can also be used when we have matched data.

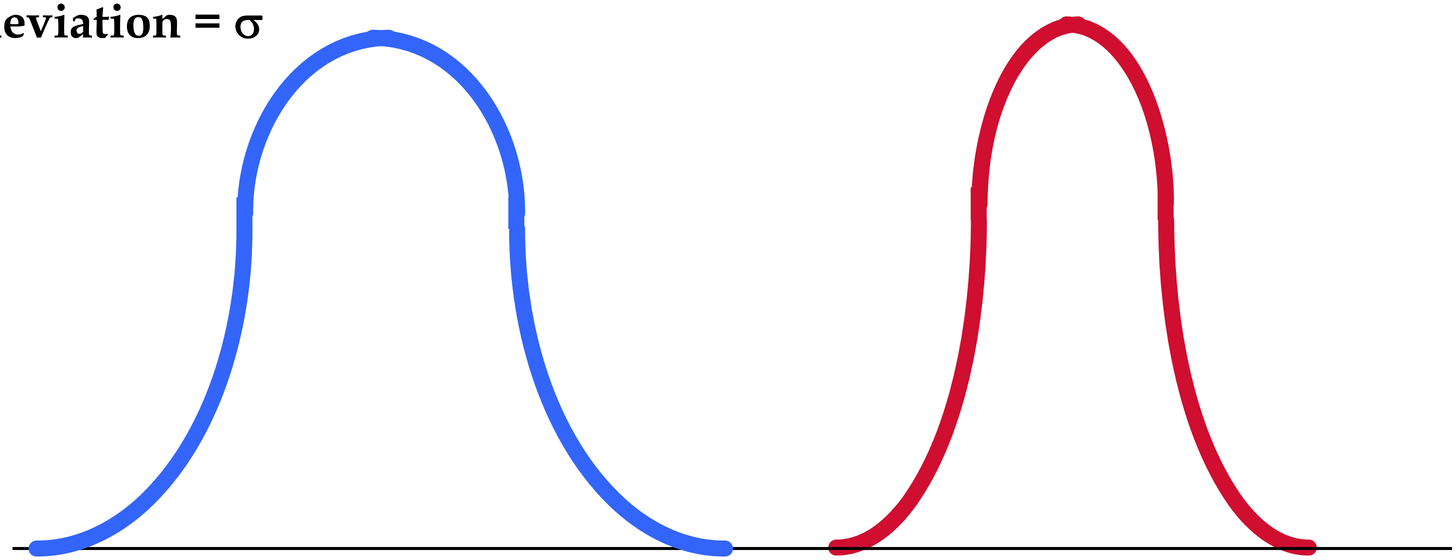
The formula will then be:

$$\frac{\bar{d} - 0}{\frac{\sigma_d}{\sqrt{n}}}$$

Testing

Single values
St. deviation = σ

Sample means
St. deviation = $\sigma / \sqrt{n}$



- But remember: The formula is in principle the same!!!!

Testing

- How do we compare two groups?
- Group 1: Mean: 157.3 cms, SD: 5.09
- Group 2: Mean: 161.3 cms, SD: 4.84

Testing

When comparing two groups:

Measured value

Hypothesis value

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - 0}{\sqrt{\frac{\sigma^2}{n_1} + \frac{\sigma^2}{n_2}}}$$

Standard deviation

But, we do not know σ!!!

Testing

When comparing two groups:

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - 0}{\sqrt{\frac{s_p^2}{n_1} + \frac{s_p^2}{n_2}}}$$

Use estimate of σ

Testing

- What is s_p????
- s_p is a weighted mean of s_1 and s_2
- Both should give a best possible guess of σ

Testing

- How to work out s_p

$$s_p = \sqrt{\frac{s_1^{\,2} \times (n_1 - 1) + s_2^{\,2} \times (n_2 - 1)}{(n_1 - 1) + (n_2 - 1)}}$$

Testing

- How to work out s_p in our example:

$$s_p = \sqrt{\frac{5.09^2 \times (16-1) + 4.84^2 \times (16-1)}{(16-1)+(16-1)}} = 4.97$$

Testing

Comparing our two groups:

$$t = \frac{(157.3 - 161.3) - 0}{\sqrt{\frac{4.97^2}{16} + \frac{4.97^2}{16}}} = -2.28$$

- The p-value is in this case 0.03

TEST TRICK I

$$\frac{(\bar{x}_1 - \bar{x}_2) - 0}{\sqrt{\frac{s_p^2}{n_1} + \frac{s_p^2}{n_2}}}$$

Choose large n's

- A small difference gives a high t-value
- A high t-value gives a low p-value
- The conclusion is a "significant" difference

TEST TRICK I

$$\frac{(\bar{x}_1 - \bar{x}_2) - 0}{\sqrt{\frac{s_p^2}{n_1} + \frac{s_p^2}{n_2}}}$$

Choose large n's

- LOOK AT ABSOLUTE DIFFERENCE AND CONFIDENCE INTERVAL

TEST TRICK II

$$\frac{(\bar{x}_1 - \bar{x}_2) - 0}{\sqrt{\frac{s_p^2}{n_1} + \frac{s_p^2}{n_2}}}$$

Choose small n's

- A large difference gives a low t-value
- A low t-value gives a high p-value
- The conclusion is a "non-significant" difference

TEST TRICK II

$$\frac{(\bar{x}_1 - \bar{x}_2) - 0}{\sqrt{\frac{s_p^2}{n_1} + \frac{s_p^2}{n_2}}}$$

Choose small n's

- LOOK AT ABSOLUTE DIFFERENCE AND CONFIDENCE INTERVAL