

IDEAL GAS LAWS

Experiments for Physics, Chemistry and Engineering Science
Using the Adiabatic Gas Law Apparatus

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Experiment 1: Ideal Gas Law

Target Students

High school and introductory college physics and chemistry.

Objectives

To show that $PV/T = nR$ for an ideal gas and determine the value of R .

Physical Principles

The well-known relationship between absolute pressure (P), volume (V), and absolute temperature (T), of an ideal gas is:

$$PV = nRT \quad (1.1)$$

where n is the number of mol in the sample and R is the ideal gas constant. For a closed system with n fixed, placing all variables on the left-hand side of the equation yields the constant value,

$$\frac{PV}{T} = nR \quad (1.2)$$

The number of mol, n , is related to the density of the gas (ρ), the molecular mass of the gas molecules (M), and the volume (V), via the relation,

$$n = \frac{\rho V}{M} \quad (1.3)$$

Procedure

A. Connections

Power the Adiabatic Gas Law unit via the AC adapter. Insert the Pressure, Volume and Temperature din connectors into channels A, B and C of the *Science Workshop* interface, respectively. Open *Data Studio* and indicate that a *voltage sensor* is connected to each channel. Adjust the *Sample Rate* to 1,000 Hz.

B. Graphical Display

Drag-and-drop the *Graph* icon from the *Displays* menu onto *Voltage, Ch A(V)* in the *Data* menu. Drag-and-drop the *Graph1* icon onto *Voltage, Ch B(V)* and then *Voltage, Ch C(V)*. In this way, graphs of Pressure, Volume and Temperature will all be displayed with a common time axis.

C. Collecting the Data

With a stopcock valve open, set the piston to the approximate middle of its range at the 10 cm mark and close both stopcocks.

Record the height of the piston at atmospheric pressure:

$$h_0 = \text{_____ cm}$$

Raise the piston to its highest position, click on *Start* and, over a span of approximately five seconds, slowly and steadily move the piston to its lowest position, then click *Stop*. Notice that the plots of voltages show the Pressure increasing as the Volume decreases.

Analysis

A. Conversions from voltages to P, V and T

Open the *Calculate* tool in *Data Studio* menu and change the *Definition* to

$$P = 100000 \cdot x \quad (\text{Pa})$$

as noted on the calibration label on the back of the Adiabatic Gas Law Apparatus and define variable "x" to be *Data Measurement* with *Voltage, ChA(V)*. Then *Accept* this calculation.

In the *Calculate* tool, click *New* and define V as the volume according to the linear calibration expression on the label. This time the variable "x" is defined as *Voltage, ChB(V)*. Then *Accept* this calculation. (If you have an older unit or wish to perform the volume calibration by hand, please refer to Appendix A.)

In the *Calculate* tool, click *New* and *Calculate* the Temperature using the linear expression on the label and defining x as the measurement from ChC.

Use the calculator to define $nR = P \cdot V / T$ and choose *Properties* to ensure that the *Numeric* display shows at least three significant figures. Plot a graph of nR by selecting the appropriate title on the vertical axis.

Use the Σ icon on the Graph menu to display the mean value of nR as well as the standard deviation.

$$nR_{\text{mean}} = \text{_____} \text{ J/K} \quad \text{Std. Dev.} = \text{_____} \text{ J/K}$$

What percentage of the mean is the standard deviation?

$$\% \text{ random error} = 100\% \cdot \text{Std. Dev.} / \text{mean} = \text{_____} \%$$

This is an estimate of the random errors associated with the experiment.

B. Number of mol, n

Calculate the volume of air (in cm^3) at the initial 10 cm height when the stopcock was open to the atmosphere using

$$V_0 = \pi r^2 h_0 = \text{_____} (\text{cm}^3)$$

where r is half the diameter displayed on the label of the Adiabatic Gas Law Apparatus.

Calculate the number of mol of gas from Eq. (1.3), with the density of air at STP of $\rho = 0.00129 \text{ g/cm}^3$, the volume V_0 , and the molecular weight of air $M_{\text{air}} = (0.8 \cdot 28 + 0.2 \cdot 32) \text{ g/mol}$ (assuming 80% nitrogen and 20% oxygen).

$$n = \frac{\rho_{\text{air}} V_0}{M_{\text{air}}} = \text{_____} \text{ mol}$$

Compute your measured value of R .

$$R_{\text{mean}} = (nR_{\text{mean}}) / n = \text{_____} \text{ J/mol} \cdot \text{K}$$

Compare your measurement with the generally accepted value of $R = 8.314 \text{ J/mol} \cdot \text{K}$.

$$\% \text{Err} = \frac{|R_{\text{mean}} - 8.314|}{8.314} \times 100\% = \text{_____} \%$$

Is this percentage greater or less than the standard deviation percentage?

Should the errors in this experiment be considered primarily random or systematic?

Experiment 2: Isothermal Processes

Experiment 2A: Boyle's Law

Target Students

High school and introductory college physics and chemistry.

Objectives

To observe that $PV = \text{const}$ for constant temperature.

Physical Principles

When a process occurs with the system in contact with a heat reservoir to keep the temperature fixed the process is considered "isothermal." For such a process, the pressure is inversely related to the volume, i.e.,

$$PV = \text{const} \quad (2.1a)$$

Since Eq. (2.1a), Boyle's Law, can be rewritten as

$$\ln(P) = -\ln(V) + \ln(\text{const}) \quad (2.2a)$$

one sees that a plot of $\ln(P)$ vs. $\ln(V)$ should be a straight line with a slope of -1 .

Procedure

Convert voltages to Pressure and Volume as in Experiment I using the *Calculator in Data Studio*. Set up a graph of P vs. V.

Follow the procedure for the ideal gas law measurements (Experiment I) but slow the *Sample* rate to one second and perform the compression **slowly** and steadily (quasi-statically) over the course of a minute-or-so. This allows the temperature of the gas to always remain at room temperature (isothermal). McNairy [1] recommends suspending a bucket from the end of the handle and easing weights into the bucket. As you collect data, you can watch the Pressure adjust to the slow change in Volume.

Analysis

- A. Use the *Calculator* to compute the $\ln(P)$ and $\ln(V)$ and make a graph of $\ln(P)$ on the y-axis vs. $\ln(V)$ on the x-axis. Perform a linear fit and record the slope:

$$\text{Slope} = \underline{\hspace{2cm}}$$

Compare this with -1.0 as expected for an inverse proportion.

- B. Since $P = nRT/V$, use the *Calculator* to obtain $1/V$ and plot P vs. $1/V$. Perform a linear fit and record the slope:

$$nRT = \text{Slope} = \underline{\hspace{2cm}}$$

Calculate the volume of air (in cm^3) at the initial 10 cm height when the stopcock was open to the atmosphere using

$$V_0 = \pi r^2 h = \underline{\hspace{2cm}} \text{ (cm}^3\text{)}$$

where r is half the diameter displayed on the label of the Adiabatic Gas Law Apparatus.

Calculate the number of mol of gas from Eq. (1.3), with the density of air at STP of $\rho = 0.00129 \text{ g/cm}^3$, the volume V_0 , and the molecular weight of air $M_{\text{air}} = (0.8 \cdot 28 + 0.2 \cdot 32) \text{ g/mol}$ (assuming 80% nitrogen and 20% oxygen).

$$n = \frac{\rho_{\text{air}} V_0}{M_{\text{air}}} = \underline{\hspace{2cm}} \text{ mol}$$

From a plot of T vs. time, use the Σ statistics tool to find the mean (room temperature),

$$T = \underline{\hspace{2cm}} \text{ K}$$

Now calculate R , $R = (nRT)/n \cdot T = (\text{slope})/n \cdot T = \underline{\hspace{2cm}}$

Compare your measurement with the generally accepted value of $R = 8.314 \text{ J/mol}\cdot\text{K}$.

$$\% \text{Err} = \frac{|R_{\text{mean}} - 8.314|}{8.314} \times 100\% = \underline{\hspace{2cm}} \%$$

Experiment 2B: Work Done by an Isothermal Process

Target Students

General physics, physics for scientists and engineers, physical chemistry, engineering thermodynamics.

Objectives

To observe that the work performed on a gas is given by

$$W = \int P dV, \text{ i.e., the area under a P vs. V curve.}$$

Physical Principles

If the compression occurs isothermally (T remains constant), the Pressure-Volume relationship is

$$P = \frac{nRT}{V} \tag{2.1b}$$

and the area under the P vs. V curve becomes,

$$W = -nRT \int_{V_i}^{V_f} \frac{1}{V} dV = nRT \ln \left(\frac{V_i}{V_f} \right) \tag{2.1b}$$

Procedure

Collect and display data as in Experiment 2A.

Analysis

A. From the plot of Pressure vs. Volume use the *Statistics* button (Σ), chose *Area* and record the area under the P vs. V curve.

Area = _____ J

- B. Calculate the number of mol of gas from the density of air at STP of $\rho = 0.00129 \text{ g/cm}^3$, the initial volume V_0 (in cm^3) and the molecular weight of air $M_{\text{air}} = (0.8 \cdot 28 + 0.2 \cdot 32) \text{ g/mol}$ (assuming 80% nitrogen and 20% oxygen).

$$n = \frac{\rho_{\text{air}} V_0}{M_{\text{air}}} = \text{_____ mol}$$

- C. Record the initial and final heights for the piston,

$$h_i = \text{_____ cm} \quad h_f = \text{_____ cm}$$

Calculate the theoretical value of the work from the ratio of the initial to final height.

$$W_{\text{theory}} = nRT \ln\left(\frac{V_i}{V_f}\right) = nRT \ln\left(\frac{h_i}{h_f}\right) = \text{_____ J}$$

$$\% \text{Err} = \frac{|\text{area} - W_{\text{theory}}|}{W_{\text{theory}}} \times 100\% = \text{_____ \%}$$

Experiment 3: Adiabatic Processes

Experiment 3A: Adiabatic Gas Law

Target Students

General physics, physics for scientists and engineers, physical chemistry, engineering thermodynamics.

Objectives

To observe the adiabatic gas law, $PV^\gamma = \text{const}$ and measure the ratio of specific heats, γ , for various gases.

Physical Principles

When a process occurs quickly enough that no heat is exchanged with the environment ($Q=0$), the process is considered "adiabatic." For such process, the relation between pressure and volume is given by

$$PV^\gamma = \text{const} \quad (3.1a)$$

Here γ is the ratio of specific heats at constant pressure and constant volume, i. e.,

$$\gamma = \frac{c_P}{c_V} \quad (3.2a)$$

A measurement of γ reveals important physical information about the structure of the gas molecules. The equipartition theorem tells us that the molar specific heat at constant volume is related to the number of degrees of freedom, f , available to each molecule through the relation

$$c_V = \frac{f}{2} R \quad (3.3a)$$

where $f = 3$ for a monatomic gas with three translational directions of motion, and $f = 5$ for a diatomic gas with three translational plus two rotational contributions to the internal energy.

The specific heat at constant pressure, c_p , is related to the specific heat at constant volume by

$$c_p = c_v + R \quad (3.4a)$$

Thus, a monatomic gas like He or Ar with $f = 3$ has the ratio $\gamma = [(3/2 + 1)R]/[(3/2)R] = 5/3$.

A diatomic gas like the N_2 and O_2 mixture found in air has $\gamma = [(5/2 + 1)R]/[(5/2)R] = 7/5$.

CO_2 has more complex degrees of freedom such that $\gamma = 1.30$.

Experimentally, you will determine γ by observing the relation between pressure and volume. Since Eq. (3.1a), the adiabatic gas law, can be rewritten as

$$\ln(P) = -\gamma \ln(V) + \ln(const) \quad (3.5a)$$

one sees that a plot of $\ln(P)$ vs. $\ln(V)$ should be a straight line with a slope of $-\gamma$.

Procedure

Follow the procedure for the Ideal Gas Law measurements (Experiment I) with two differences:

- A. Insert a pin in the lower hole as an abrupt stop for the piston motion.
- B. The gas should be compressed as rapidly as possible to make the experiment approximately adiabatic.

Analysis

- A. Convert voltages to Pressure and Volume as in Experiment I.
- B. Use the Calculator to compute the $\ln(P)$ and $\ln(V)$ and make a graph of $\ln(P)$ on the y-axis vs. $\ln(V)$ on the x-axis. Perform a linear fit and record the slope:

$$\text{Slope} = \gamma = \underline{\hspace{2cm}}$$

For air, compare this with 1.40.

Repeat the experiment using argon (or helium) and CO_2 comparing with the respective ratios for these gases.

Experiment 3B: Work Done by an Adiabatic Process

Target Students

Advanced high school, introductory college physics and chemistry, physics for scientists and engineers, physical chemistry, engineering thermodynamics.

Objectives

To observe that the work performed on a gas is given by

$$W = \int P dV$$

, i.e., the area under a P vs. V curve.

Physical Principles

When a gas is compressed under constant pressure, the work performed on the systems is simply the product of force times displacement or,

$$W = -F\Delta y = -PA\Delta y = -P\Delta V \quad (3.1b)$$

If the pressure changes as the compression occurs, the work done on the gas becomes the integral

$$W = -\int_{V_i}^{V_f} P dV \quad (3.2b)$$

corresponding to the negative of the area under a Pressure vs. Volume curve.

If the compression occurs adiabatically, there is no heat exchange with the environment and by the First Law of Thermodynamics the change in internal energy will equal the work performed on the gas,

$$\Delta U = Q + W = W \quad (3.3b)$$

Thus, when the gas is compressed adiabatically, we can measure the area under the P vs. V curve and compare with the change in internal energy given by,

$$\Delta U = \frac{f}{2} nR\Delta T \quad (3.4b)$$

for an ideal gas with f degrees of freedom.

For the diatomic molecules of air (f = 5) this becomes,

$$Area = \frac{5}{2} nR\Delta T \quad (3.5b)$$

One can also analytically calculate the area under the P vs. V curve by noting

$$W = -\int_{V_i}^{V_f} PdV = -k \int_{V_i}^{V_f} \frac{dV}{V^\gamma} = -k \left[\frac{V^{1-\gamma}}{1-\gamma} \right]_{V_i}^{V_f} = \frac{P_i V_i^\gamma}{\gamma-1} (V_f^{1-\gamma} - V_i^{1-\gamma}) = \frac{P_i V_i}{\gamma-1} (r^{\gamma-1} - 1) \quad (3.6b)$$

where $r = V_i/V_f$ is the compression ratio.

Procedure

Follow the procedure for the adiabatic gas law measurements (Experiment 3A).

Analysis

- Convert voltages to Pressure, Volume and Temperature using the *Data Studio Calculator* as outlined in Experiment 3A.
- Construct a plot of Pressure vs. Volume and expand the view appropriately. Using the *Statistics* button (Σ), chose *Area* and record the area under the P vs. V curve. (**Be careful** to highlight data that avoids both ends of the P vs. V curve! The numerical integration breaks down when the ends are included.)

$$Area = \text{_____} \text{ J}$$

- Plot Temperature vs. time and use the statistics to record the minimum and maximum temperature and compute the change in temperature.

$$T_{\min} = \text{_____} \text{ K}$$

$$T_{\max} = \text{_____} \text{ K}$$

$$\Delta T = \text{_____} \text{ K}$$

D. Number of mol, n.

Calculate the volume of air (in cm³) at the initial 10 cm height when the stopcock was open to the atmosphere using

$$V_0 = \pi r^2 h_0 = \text{_____} \text{ (cm}^3\text{)}$$

where r is half the diameter displayed on the label of the Adiabatic Gas Law Apparatus.

Calculate the number of mol of gas from Eq. (1.3), with the density of air at STP of $\rho = 0.00129 \text{ g/cm}^3$, the volume V_0 , and the molecular weight of air $M_{\text{air}} = (0.8 \cdot 28 + 0.2 \cdot 32) \text{ g/mol}$ (assuming 80% nitrogen and 20% oxygen).

$$n = \frac{\rho_{\text{air}} V_0}{M_{\text{air}}} = \text{_____} \text{ mol}$$

Compute the change in internal energy of the air,

$$\Delta U = \frac{5}{2} n R \Delta T = \text{_____} \text{ J}$$

Compare the change in internal energy with the area under the P vs. V curve.

Compare the area under the curve with the prediction of Eq. 3.6b.

Experiment 4: Complete Cycle

Target Students

General physics, physics for scientists and engineers, physical chemistry, engineering thermodynamics.

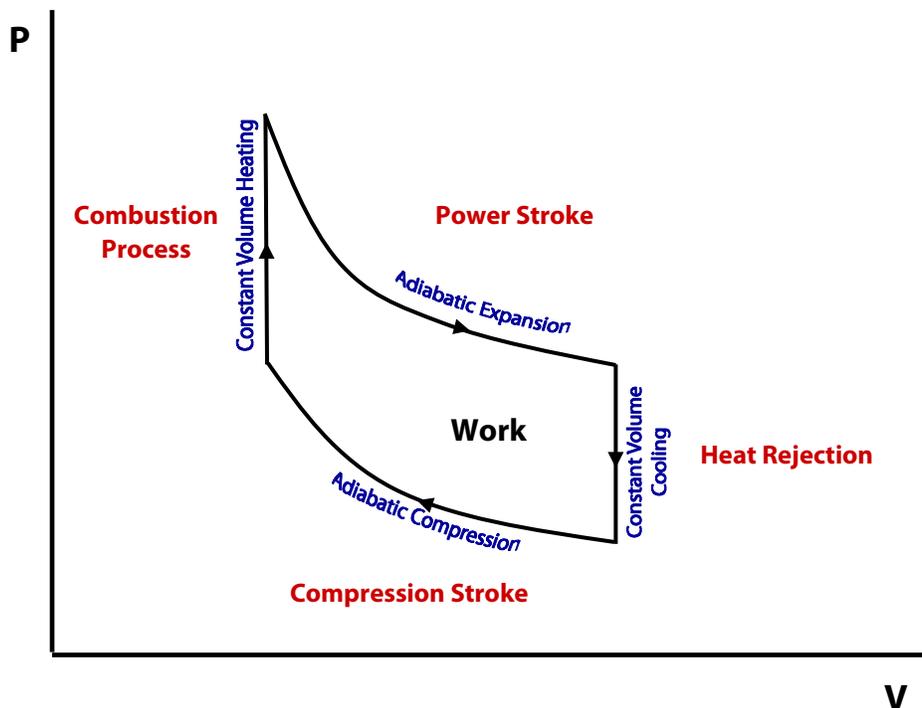
Objectives

Observe the Otto cycle operating in reverse and compare the ratio of the (work on the gas)/ (heat out of gas) with the Otto cycle efficiency.

Physical Principles

The ideal Otto cycle models the action of a four-stroke gasoline engine and includes the strokes shown on the PV diagram below:

Ideal Otto Cycle
P-V diagram



The efficiency of any heat engine is defined as,

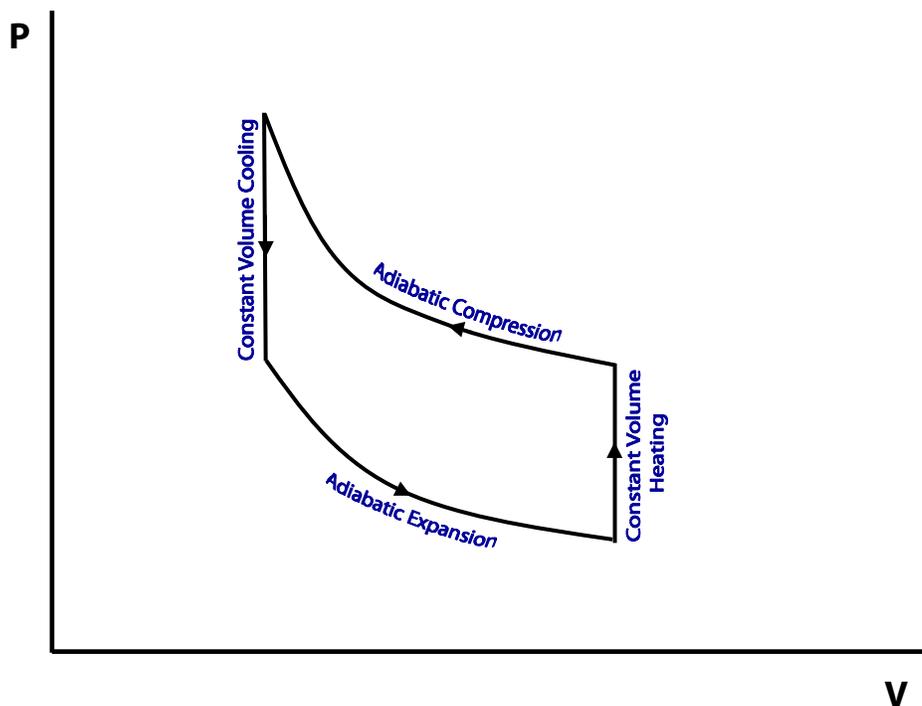
$$\eta = \frac{\text{Work Out}}{\text{Heat In}} \quad (4.1)$$

This is equivalent to the net area under a P vs. V curve and divided by the heat entering the system. For the Otto cycle, the work is area between the two adiabatic strokes and heat enters the system during the isovolumetric combustion stroke.

In this experiment, you will be operating the Otto Cycle in reverse.

Reversed Otto Cycle

P-V diagram



The procedure begins with an adiabatic compression. The area under the upper-most curve is the work done on the gas, W_{on} . The gas is then cooled to the original temperature isovolumetrically, such that $Q_{\text{out}} = W_{\text{on}}$. An adiabatic expansion is then followed by isovolumetric heating with $Q_{\text{in}} = W_{\text{by}}$.

Though the process is not a heat engine, we can compare the net work done on the gas with the heat flow out,

$$\xi = \frac{W_{net}}{Q_{out}} = \frac{W_{on} - W_{by}}{W_{on}} \quad (4.2)$$

And compare the result with the Otto cycle efficiency which is known to be

$$\eta_{Otto} = 1 - \frac{1}{r^{\gamma-1}} \quad (4.3)$$

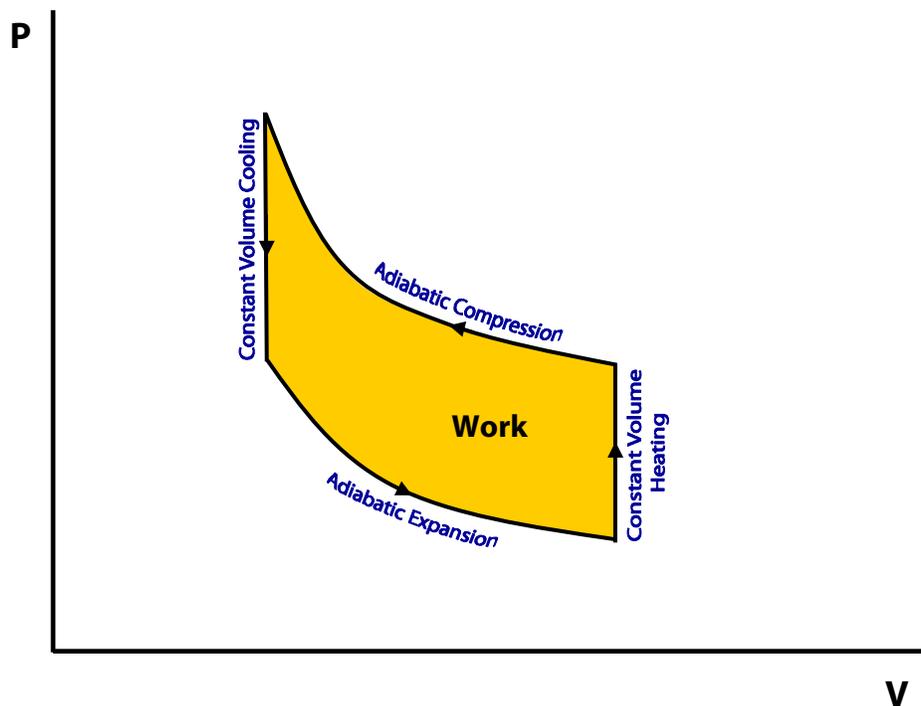
where r is the compression ratio

$$r = \frac{V_{max}}{V_{min}} \quad (4.4)$$

and γ is the ratio of specific heats at constant pressure and constant volume (for air, $\gamma = 1.4$).

Reversed Otto Cycle

P-V diagram



Procedure

Set up the experiment as in Experiment 3 (Adiabatic Gas Law), converting voltages into Pressure, Volume and Temperature. Set up a plot of Pressure vs. Volume. Put the two pins in the device to limit the motion of the handle. Raise the piston, open the stopcock and allow the cylinder to come to equilibrium with its surroundings. Close the stopcock. Once you *Start* the data collection, perform the following four steps before you *Stop*:

1. Quickly compress the piston in an adiabatic compression stroke, continuing to hold the piston down at the bottom of the stroke. (Stroke 5 to 4 in the Otto cycle diagram).
2. Hold the piston down while an isovolumetric (isochoric) transfer of heat out of the system takes place. Continue to hold the piston down until you observe that the pressure is no longer dropping (approximately ½ minute). (Stroke 4 to 3 in the diagram).
3. Quickly raise the piston in an adiabatic expansion and hold up at the top of the stroke (Stroke 3 to 6 in the diagram).
4. Continue holding the piston up until the cycle on the P-V curve is completed. (Stroke 6 to 5 in the diagram).

Analysis

- A. Retake the adiabatic compression stroke only while examining the plot of P vs. V. Use the Σ statistics button to evaluate the area under this high-pressure, adiabatic curve. (For numerical reasons, **avoid the end points** of the P vs. V curve!)

$$W_{on} = \text{area}_{upper} = \underline{\hspace{2cm}} \text{ J}$$

- B. Retake the adiabatic expansion stroke and measure the area under this lower pressure curve. (Again avoiding the end points).

$$W_{by} = \text{area}_{lower} = \underline{\hspace{2cm}} \text{ J}$$

- C. Calculate the ratio,

$$\xi = \frac{W_{on} - W_{by}}{W_{on}} = \underline{\hspace{2cm}}$$

D. Compute the compression ratio from the ratio of the maximum to minimum cylinder heights,

$$r = \frac{V_{\max}}{V_{\min}} = \frac{h_{\max}}{h_{\min}} = \underline{\hspace{2cm}}$$

Calculate the theoretical efficiency of an Otto cycle as

$$\eta_{\text{Otto}} = 1 - \frac{1}{r^{\gamma-1}} = \underline{\hspace{2cm}}$$

Compare with the measured ratio for the Otto cycle in reverse,

$$\xi = \frac{W_{\text{on}} - W_{\text{by}}}{W_{\text{on}}}$$

You can remove the pins and investigate what happens to the efficiency and the ratio, ξ , with a larger values of compression ratio, r .

Calculate the ideal efficiency of a gasoline engine where the compression ratio is normally less than 10:1.

Calculate the ideal efficiency for a diesel engine (assuming that it operates under the Otto cycle) where the compression ratio is often greater than 22:1.

Experiment 5: Heat Capacity of Gas from Pressure, Volume, and Temperature Data

Target Students

Physics for scientists and engineers, physical chemistry, engineering thermodynamics.

Objectives

To illustrate how the abstract mathematics of differential equations can be utilized to obtain a concrete thermodynamic quantity.

Physical Principles

In physical chemistry the thermodynamic calculations of material properties can be made if heat capacities of the materials are available. The heat capacity of a gas under constant pressure, C_p , defined as $(\partial V/\partial T)_p$, can be derived as $C_p = T(\partial V/\partial T)_p(\partial P/\partial T)_s$. By following the same principle for the measurement of the heat capacity of a liquid as described in Reference 1, analogous measurements to determine the heat capacity of a gas can also be made.

$(\partial V/\partial T)_p$ and $(\partial P/\partial T)_s$ can be reasonably approximated by finite changes $\Delta V/\Delta T$ and $\Delta P/\Delta T$, respectively. The change in volume of a gas with temperature can be measured using standard equipment such as a temperature-controlled water bath and a water manometer. The change in pressure with temperature requires an apparatus which permits a fast measurement with negligible heat loss during the change. The Adiabatic Gas Law Apparatus was chosen because with it a gas could be compressed quickly, and the signals from the temperature and pressures sensors could be recorded simultaneously.

Procedure

1. Flush the chamber with the sample gas for about 2 minutes. Move the lever up and down during the flushing to ensure thorough mixing and elimination from the chamber of air or any previously used gas.

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2. Close all the valves and hold the lever at the highest position. Set the computer interface (Science Workshop, PASCO) to record temperature on the x-axis and pressure on the y-axis. Initiate computer acquisition and push the lever down quickly for a rapid compression of the gas in the chamber.
 3. Use the curve-fit program in the software to determine the slope of the displayed trace of pressure vs. temperature. This will be the experimental value for $\Delta P/\Delta T$.

Calculations

1. Multiply the $\Delta P/\Delta T$ by 2.29^\dagger to convert from the voltage slope to kPa/K, and then multiply by 0.00986 to obtain the unit of atm/K. This calculated $\Delta P/\Delta T$ will be approximately $(\partial P/\partial T)_s$ and can be combined with the $\Delta V/\Delta T$ $(\partial V/\partial T)_p$ from another experiment to determine C_p .
2. Read the height (cm) of the chamber and calculate its volume by
$$V \text{ (cc)} = \text{height} \cdot \Pi \cdot (4.51/2)^2$$
3. Use the perfect gas equation ($PV=nRT$), the calculated chamber volume, the ambient temperature, and the barometric pressure of the day to estimate the number of mole of the gas in the chamber.
4. Report C_p in j/K/mol.

Reliability of Results

The calculated results from experiments over many years have been comparable to literature values. The gases used have been argon and carbon dioxide. For argon, the average experimentally determined C_p from the years 1995 to 2006 is 31 j/K/mol, while the literature value is 20.8 j/K/mol. For carbon dioxide, the average over the same period of time is 41 j/K/mol, and the literature value is 37 j/K/mol. The typical uncertainty in each reported result has been between 10 and 20 percent.

[†] Pressure and temperature calibrations of the Adiabatic Gas Law Apparatus.

Appendix A: Calibration of V

Necessary for older Adiabatic Gas Law Apparatus models, optional exercise for newer models

If you have an older unit without the volume calibration, the following procedure will enable you to determine the Volume/Voltage relation.

Note the diameter of the cylinder printed on the unit and record the radius,
 $r =$ _____ cm = _____ meters.

Drag-and-drop the *Digits* icon from the *Displays* menu onto the *Voltage, Ch B(V)* in the *Data* menu. Open one of the stopcock valves, click the *Start* button and record the height (h) of the bottom of the piston and corresponding volume voltage, V_{vol} for each centimeter over the range of motion (approximately 6–15 cm).

Table I: Cylinder height vs. Voltage

V_{vol} (Volts)	h (m)
	0.06
	0.07
	0.08
	0.09
	0.10
	0.11
	0.12
	0.13
	0.14
	0.15

Close the *Digits* display.

Use your favorite linear regression method to determine the slope and intercept of this straight-line relationship.

$$h = \text{_____} \times V_{vol} + \text{_____} \text{ (meters)}$$

This becomes a relationship between Volume and Voltage using

$$\text{Volume} = \pi r^2 h = \text{_____} \times V_{vol} + \text{_____} \text{ (m}^3\text{)}$$

Appendix B: Additional Analysis to Improve Adiabatic Gas Law Results

A careful examination of the adiabatic data (Experiment 3A) shows a slight time delay between the piston reaching the minimum volume and the pressure achieving its maximum. The amount of delay is on the order of the height of the cylinder divided by the speed of sound. You can adjust the pressure to account for this delay. Drag-and-drop the *Graph* icon onto P and the *Graph1* icon onto V so that both Pressure and Volume are displayed on the same scale. Moving the cursor to the numbers on the horizontal scale and sliding allows you to expand the view until the small time delay between the abrupt minimum in the Volume and the slightly delayed maximum in Pressure becomes notable. Use the XY cross hair tool to record the two times to within a millisecond:

$$t_{\min \text{ Vol}} = \text{_____ S} \quad t_{\max \text{ P}} = \text{_____ S}$$

Calculate the pressure time delay,

$$\Delta t = t_{\max \text{ P}} - t_{\min \text{ Vol}} = \text{_____ s (should be a few milliseconds)}$$

Use the Calculator to compute the corrected pressure assuming the first order in a Taylor series expansion of the pressure with time,

$$P_{\text{corr}} = P + \frac{dP}{dt} \cdot \Delta t \tag{3.6a}$$

Note that derivatives are available under the *Special* functions found in the *Calculator*. Use the *Calculator* to compute the $\ln(P_{\text{corr}})$ and $\ln(V)$ and make a graph of $\ln(P_{\text{corr}})$ on the y-axis vs. $\ln(V)$ on the x-axis. Perform a linear fit and record the slope:

$$\text{Slope} = \gamma = \text{_____}$$

Has the introduction of pressure time delay into the analysis improved your results?

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2. Gill, Stanley J.; West, Earl M. The Indirect Determination of Heat Capacity, C_p of a Liquid, *Journal of Chemical Education*, vol. 43, 557–559 (1966).