7.1 Apply the Pythagorean Theorem

Goal • Find side lengths in right triangles.

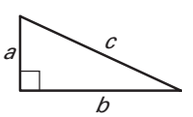
Your Notes

VOCABULARY

Pythagorean triple A Pythagorean triple is a set of three positive integers a, b, and c that satisfy the equation $c^2 = a^2 + b^2$.

THEOREM 7.1: PYTHAGOREAN THEOREM

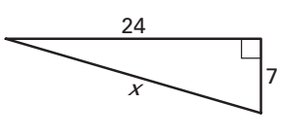
In a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs.



$c^2 = a^2 + b^2$

Example 1 ***Find the length of a hypotenuse***

Find the length of the hypotenuse of the right triangle.



Solution

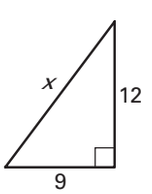
$(\text{hypotenuse})^2 = (\text{leg})^2 + (\text{leg})^2$		**Pythagorean Theorem**
$x^2 =$	$7^2 + 24^2$	**Substitute.**
$x^2 =$	$49 + 576$	**Multiply.**
$x^2 =$	625	**Add.**
$x =$	25	**Find the positive square root.**

In the equation for the Pythagorean Theorem, "length of hypotenuse" and "length of leg" was shortened to "hypotenuse" and "leg".

✔ ***Checkpoint*** **Complete the following exercise.**

1. Find the length of the hypotenuse of the right triangle.

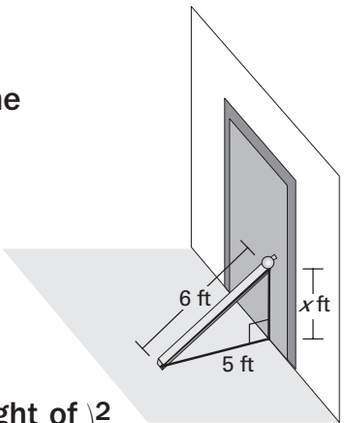
$x = 15$



Your Notes

Example 2 ***Find the length of a leg***

Door A 6 foot board rests under a doorknob and the base of the board is 5 feet away from the bottom of the door. Approximately how high above the ground is the doorknob?



Solution

$\left(\begin{matrix}\text{Length}\\ \text{of board}\end{matrix}\right)^2 = \left(\begin{matrix}\text{Distance}\\ \text{from door}\end{matrix}\right)^2 + \left(\begin{matrix}\text{Height of}\\ \text{doorknob}\end{matrix}\right)^2$

$6^2 = 5^2 + x^2$	**Substitute.**
$36 = 25 + x^2$	**Multiply.**
$11 = x^2$	**Subtract 25 from each side.**
$\sqrt{11} = x$	**Find positive square root.**
$3.3 \approx x$	**Approximate with a calculator.**

The board is resting against the doorknob at about 3.3 feet above the ground.

In real-world applications, it is usually appropriate to use a calculator to approximate the square root of a number. Round your answer to the nearest tenth.

✔ ***Checkpoint*** **Complete the following exercise.**

2. A 5 foot board rests under a doorknob and the base of the board is 3.5 feet away from the bottom of the door. Approximately how high above the ground is the doorknob?

about 3.6 feet

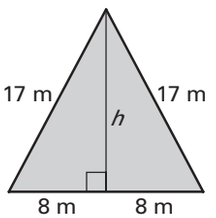
Your Notes

Example 3 ***Find the area of an isosceles triangle***

Find the area of the isosceles triangle with side lengths 16 meters, 17 meters, and 17 meters.

Solution

Step 1 **Draw** a sketch. By definition, the length of an altitude is the height of the triangle. In an isosceles triangle, the altitude to the base is also a perpendicular bisector. So, the altitude divides the triangle into two right triangles with the dimensions shown.



Step 2 **Use** the Pythagorean Theorem to find the height of the triangle.

$c^2 = a^2 + b^2$ **Pythagorean Theorem**

$17^2 = 8^2 + h^2$ **Substitute.**

$289 = 64 + h^2$ **Multiply.**

$225 = h^2$ **Subtract 64 from each side.**

$15 = h$ **Find the positive square root.**

Step 3 **Find** the area.

$$\text{Area} = \frac{1}{2}(\text{base})(\text{height}) = \frac{1}{2}(16)(15) = 120$$

The area of the triangle is 120 square meters.

You may find it helpful to memorize the basic Pythagorean triples, shown in **bold**, for standardized tests.

COMMON PYTHAGOREAN TRIPLES AND SOME OF THEIR MULTIPLES

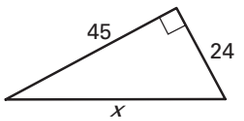
3, 4, 5	**5, 12, 13**	**8, 15, 17**	**7, 24, 25**
6, 8, 10	10, 24, 26	16, 30, 34	14, 48, 50
9, 12, 15	15, 36, 39	24, 45, 51	21, 72, 75
30, 40, 50	50, 120, 130	80, 150, 170	70, 240, 250
$3x$, $4x$, $5x$	$5x$, $12x$, $13x$	$8x$, $15x$, $17x$	$7x$, $24x$, $25x$

The most common Pythagorean triples are in bold. The other triples are the result of multiplying each integer in a bold face triple by the same factor.

Your Notes

Example 4 ***Find length of a hypotenuse using two methods***

Find the length of the hypotenuse of the right triangle.



Solution

Method 1: Use a Pythagorean triple.

A common Pythagorean triple is 8, 15, 17. Notice that if you multiply the lengths of the legs of the Pythagorean triple by 3, you get the lengths of the legs of this triangle: $8 \cdot 3 = 24$ and $15 \cdot 3 = 45$. So, the length of the hypotenuse is $17 \cdot 3 = 51$.

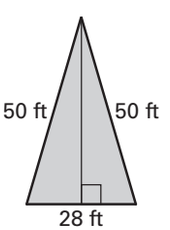
Method 2: Use the Pythagorean Theorem.

$x^2 = 24^2 + 45^2$	**Pythagorean Theorem**
$x^2 = 576 + 2025$	**Multiply.**
$x^2 = 2601$	**Add.**
$x = 51$	**Find the positive square root.**

✔ ***Checkpoint*** **Complete the following exercises.**

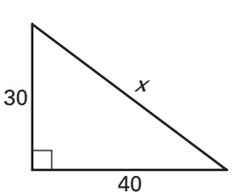
3. Find the area of the triangle.

672 ft^2



4. Use a Pythagorean triple to find the unknown side length of the right triangle.

50



Homework