

7.3

Use Similar Right Triangles

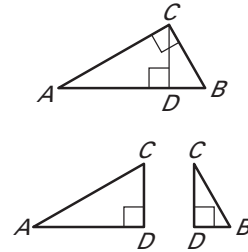
Goal • Use properties of the altitude of a right triangle.

Your Notes

THEOREM 7.5

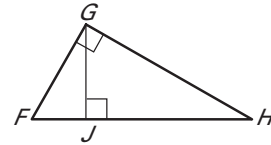
If the altitude is drawn to the hypotenuse of a right triangle, then the two triangles formed are similar to the original triangle and to each other.

$\triangle CBD \sim \triangle ABC$, $\triangle ACD \sim \triangle ABC$,
and $\triangle CBD \sim \triangle ACD$.



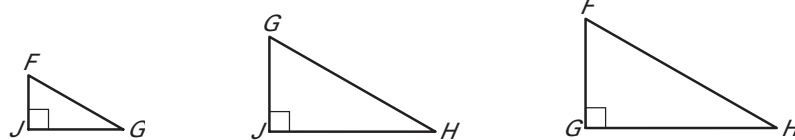
Example 1 Identify similar triangles

Identify the similar triangles in the diagram.



Solution

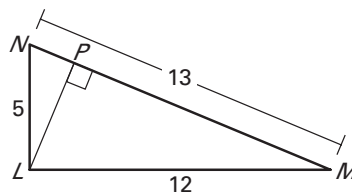
Sketch the three similar right triangles so that the corresponding angles and sides have the same orientation.



$\triangle FJG \sim \triangle GJH \sim \triangle FGH$

Checkpoint Complete the following exercise.

1. Identify the similar triangles in the diagram.

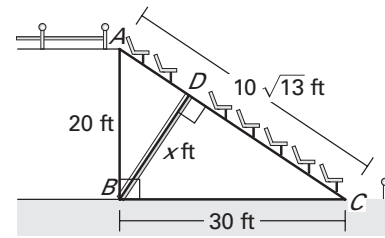


$\triangle NPL \sim \triangle LPM \sim \triangle NLM$

Your Notes

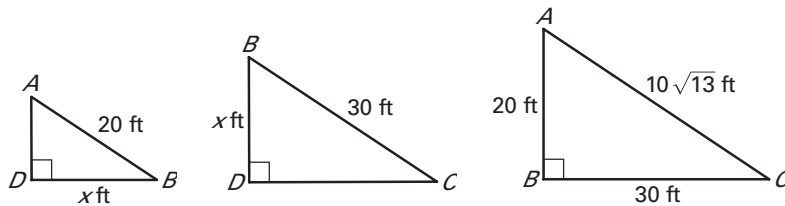
Example 2 Find the length of the altitude to the hypotenuse

Stadium A cross section of a group of seats at a stadium shows a drainage pipe \overline{BD} that leads from the seats to the inside of the stadium. What is the length of the pipe?



Solution

Step 1 Identify the similar triangles and sketch them.



$$\triangle ADB \sim \triangle BDC \sim \triangle ABC$$

Step 2 Find the value of x . Use the fact that $\triangle BDC \sim \triangle ABC$ to write a proportion.

$$\frac{BD}{AB} = \frac{BC}{AC}$$

$$\frac{x}{20} = \frac{30}{10\sqrt{13}}$$

$$(10\sqrt{13})x = 20(30) \quad \text{Cross Products Property}$$

$$x \approx 16.6 \quad \text{Approximate.}$$

Corresponding side lengths of similar triangles are in proportion.

Substitute.

Cross Products Property

Approximate.

The length of the pipe is about 16.6 feet.

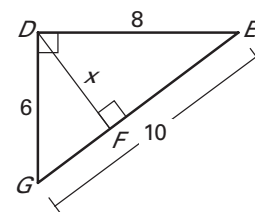
Notice that if you tried to write a proportion using $\triangle ADB$ and $\triangle BDC$, there would be two unknowns, so you would not be able to solve for x .

✓ **Checkpoint** Complete the following exercise.

2. Identify the similar triangles. Then find the value of x .

$$\triangle GFD \sim \triangle DFE \sim \triangle GDE;$$

$$x = 4.8$$

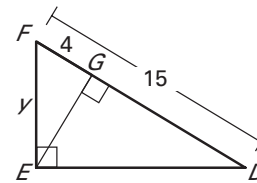


Your Notes

Notice that $\triangle FEG$ and $\triangle FDE$ both contain the side with length y , so these are the similar pair of triangles to use to solve for y .

Example 3 Use a geometric mean

Find the value of y . Write your answer in simplest radical form.



Solution

Write a proportion.

$$\frac{\text{length of hyp. of } \triangle FDE}{\text{length of hyp. of } \triangle FEG} = \frac{\text{length of shorter leg of } \triangle FDE}{\text{length of shorter leg of } \triangle FEG}$$

$$\frac{15}{y} = \frac{y}{4}$$

Substitute.

$$\frac{60}{y} = y^2$$

Cross Products Property

$$\sqrt{60} = y$$

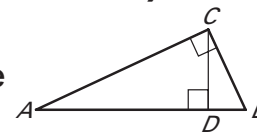
Take positive square roots.

$$2\sqrt{15} = y$$

Simplify.

THEOREM 7.6: GEOMETRIC MEAN (ALTITUDE) THEOREM

In a right triangle, the altitude from the right angle to the hypotenuse divides the hypotenuse into two segments.

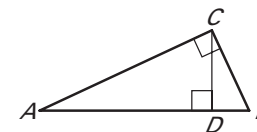


$$\frac{BD}{CD} = \frac{CD}{AD}$$

The length of the altitude is the geometric mean of the lengths of the two segments.

THEOREM 7.7: GEOMETRIC MEAN (LEG) THEOREM

In a right triangle, the altitude from the right angle to the hypotenuse divides the hypotenuse into two segments.



$$\frac{AB}{CB} = \frac{CB}{DB} \text{ and}$$

$$\frac{AB}{AC} = \frac{AC}{AD}$$

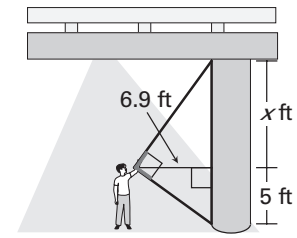
The length of each leg of the right triangle is the geometric mean of the lengths of the hypotenuse and the segment of the hypotenuse that is adjacent to the leg.

Your Notes

Example 4 Find a height using indirect measurement

Overpass To find the clearance under an overpass, you need to find the height of a concrete support beam.

You use a cardboard square to line up the top and bottom of the beam. Your friend measures the vertical distance from the ground to your eye and the distance from you to the beam. Approximate the height of the beam.



Solution

By Theorem 7.6, you know that 6.9 is the geometric mean of x and 5.

$$\frac{x}{6.9} = \frac{6.9}{5} \quad \text{Write a proportion.}$$

$$x \approx \underline{9.5} \quad \text{Solve for } x.$$

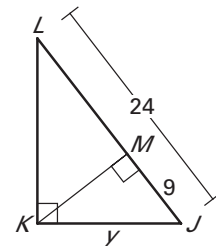
So, the clearance under the overpass is

$$5 + x \approx 5 + \underline{9.5} = \underline{14.5} \text{ feet.}$$

✓ Checkpoint Complete the following exercises.

3. Find the value of y . Write your answer in simplest radical form.

$$y = 6\sqrt{6}$$



4. The distance from the ground to Larry's eyes is 4.5 feet. How far from the beam in Example 4 would he have to stand in order to measure its height?

about 6.7 feet

Homework