7.4 Special Right Triangles

Goal • Use the relationships among the sides in special right triangles.

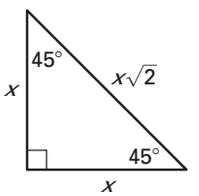
Your Notes

The extended ratio of the side lengths of a 45°-45°-90° triangle is $1:1:\sqrt{2}$.

THEOREM 7.8: 45°-45°-90° TRIANGLE THEOREM

In a 45°-45°-90° triangle, the hypotenuse is $\sqrt{2}$ times as long as each leg.

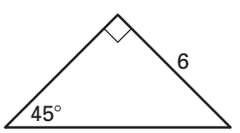
hypotenuse = leg • $\sqrt{2}$



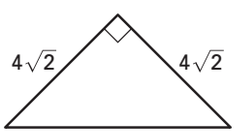
Example 1 *Find hypotenuse length in a 45°-45°-90° triangle*

Find the length of the hypotenuse.

a.



b.



Solution

a. By the Triangle Sum Theorem, the measure of the third angle must be 45°. Then the triangle is a 45° - 45° -90° triangle, so by Theorem 7.8, the hypotenuse is $\sqrt{2}$ times as long as each leg.

hypotenuse = leg • $\sqrt{2}$ — 45° - 45° -90° Triangle Theorem

= $6\sqrt{2}$ — Substitute.

b. By the Base Angles Theorem and the Corollary to the Triangle Sum Theorem, the triangle is a 45°-45°-90° triangle.

Remember the following properties of radicals:
$\sqrt{a} \cdot \sqrt{b} = \sqrt{a \cdot b}$;
$\sqrt{a} \cdot \sqrt{a} = a$

hypotenuse = leg • $\sqrt{2}$ — 45°-45°-90° Triangle Theorem

= $4\sqrt{2} \cdot \sqrt{2}$ — Substitute.

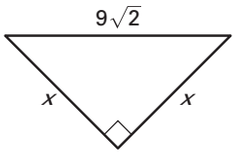
= $4 \cdot 2$ — Product of square roots

= 8 — Simplify.

Your Notes

Example 2 ***Find leg lengths in a 45°-45°-90° triangle***

Find the lengths of the legs in the triangle.



Solution

By the Base Angles Theorem and the Corollary to the Triangle Sum Theorem, the triangle is a 45°-45°-90° triangle.

hypotenuse = leg • $\sqrt{2}$ **45°-45°-90° Triangle Theorem**

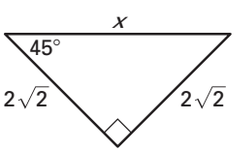
$9\sqrt{2} = x \cdot \sqrt{2}$ **Substitute.**

$\frac{9\sqrt{2}}{\sqrt{2}} = \frac{x\sqrt{2}}{\sqrt{2}}$ **Divide each side by $\sqrt{2}$.**

$9 = x$ **Simplify.**

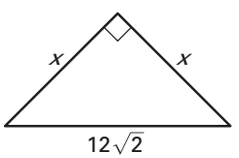
✔ ***Checkpoint*** **Find the value of the variable.**

1.



$x = 4$

2.



$x = 12$

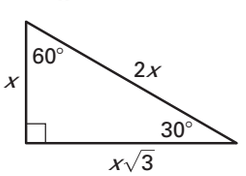
The extended ratio of the side lengths of a 30°-60°-90° triangle is $1 : \sqrt{3} : 2$.

THEOREM 7.9: 30°-60°-90° TRIANGLE THEOREM

In a 30°-60°-90° triangle, the hypotenuse is twice as long as the shorter leg, and the longer leg is $\sqrt{3}$ times as long as the shorter leg.

hypotenuse = 2 • shorter leg

longer leg = shorter leg • $\sqrt{3}$



Your Notes

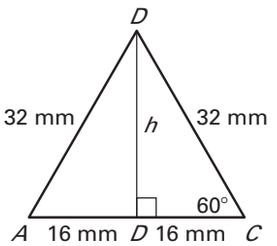
Example 3 *Find the height of an equilateral triangle*

Music You make a guitar pick that resembles an equilateral triangle with side lengths of 32 millimeters. What is the approximate height of the pick?

Remember that in an equilateral triangle, the altitude to a side is also the median to that side. So, altitude $\overline{BD}$ bisects $\overline{AC}$.

Solution

Draw the equilateral triangle described. Its altitude forms the longer leg of two 30° - 60° -90° triangles. The length h of the altitude is approximately the height of the pick.

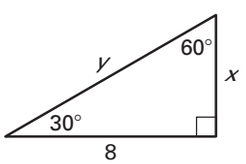


longer leg = shorter leg • $\sqrt{3}$

h = 16 • $\sqrt{3}$ ≈ 27.7 mm

Example 4 *Find lengths in a 30°-60°-90° triangle*

Find the values of x and y. Write your answer in simplest radical form.



Solution

Step 1 **Find** the value of x.

longer leg = shorter leg • $\sqrt{3}$

$8 = x\sqrt{3}$ **Substitute.**

$\frac{8}{\sqrt{3}} = x$ **Divide each side by** $\sqrt{3}$.

$\frac{8}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = x$ **Multiply numerator and denominator by** $\sqrt{3}$.

$\frac{8\sqrt{3}}{3} = x$ **Multiply fractions.**

Step 2 **Find** the value of y.

hypotenuse = 2 • shorter leg

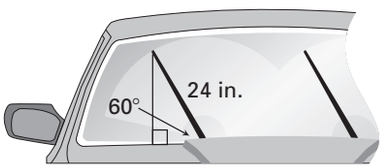
$y = 2 \cdot \frac{8\sqrt{3}}{3} = \frac{16\sqrt{3}}{3}$

Your Notes

Example 5 ***Find a height***

Windshield wipers A car is turned off while the windshield wipers are moving. The 24 inch wipers stop, making a 60° angle with the bottom of the windshield. How far from the bottom of the windshield are the ends of the wipers?

Solution



The distance d is the length of the longer leg of a 30° - 60° -90° triangle.

The length of the hypotenuse is 24 inches.

hypotenuse = 2 • shorter leg — 30° - 60° -90° Triangle Theorem

24 = 2 • s — Substitute.

12 = s — Divide each side by 2.

longer leg = shorter leg • $\sqrt{3}$ — 30° - 60° -90° Triangle Theorem

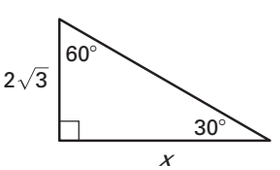
$d = 12\sqrt{3}$ — Substitute.

$d \approx 20.8$ — Approximate.

The ends of the wipers are about 20.8 inches from the bottom of the windshield.

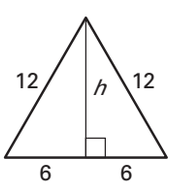
✔ ***Checkpoint*** **In Exercises 3 and 4, find the value of the variable.**

3.



$x = 6$

4.



$h = 6\sqrt{3}$

Homework

5. In Example 5, how far from the bottom of the windshield are the ends of the wipers if they make a 30° angle with the bottom of the windshield?

12 inches