7.5 Apply the Tangent Ratio

Goal • Use the tangent ratio for indirect measurement.

Your Notes

VOCABULARY

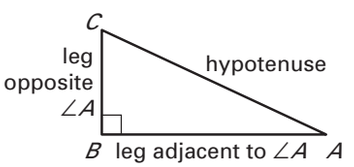
Trigonometric ratio A trigonometric ratio is a ratio of the lengths of two sides in a right triangle.

Tangent The ratio of the lengths of the legs in a right triangle is called the tangent of the angle.

Remember these abbreviations: tangent → tan opposite → opp. adjacent → adj.

TANGENT RATIO

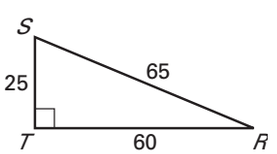
Let $\triangle ABC$ be a right triangle with acute $\angle A$. The tangent of $\angle A$ (written as tan A) is defined as follows:



$$\tan A = \frac{\text{length of leg opposite } \angle A}{\text{length of leg adjacent to } \angle A} = \frac{BC}{AC}$$

Example 1 *Find tangent ratios*

Find tan S and tan R. Write each answer as a fraction and as a decimal rounded to four places, if necessary.



Unless told otherwise, round values of trigonometric ratios to the ten-thousandths' place and round lengths to the tenths' place.

Solution

$$\tan S = \frac{\text{opp. } \angle S}{\text{adj. to } \angle S} = \frac{RT}{ST} = \frac{60}{25} = \frac{12}{5} = 2.4$$

$$\tan R = \frac{\text{opp. } \angle R}{\text{adj. to } \angle R} = \frac{ST}{RT} = \frac{25}{60} = \frac{5}{12} \approx 0.4167$$

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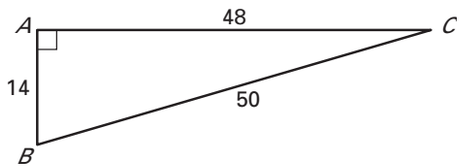
Your Notes

✔ ***Checkpoint*** **Find tan *B* and tan *C*. Write each answer as a fraction and as a decimal rounded to four places.**

1.

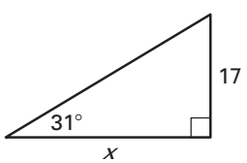


$\tan B = \frac{24}{7} \approx 3.4286$, $\tan C = \frac{7}{24} \approx 0.2917$

Example 2 ***Find a leg length***

Find the value of *x*.

Use the tangent of an acute angle to find a leg length.



$\tan 31° = \frac{\text{opp.}}{\text{adj.}}$ — **Write ratio for tangent of 31°.**

$\tan 31° = \frac{17}{x}$ — **Substitute.**

$x \cdot \tan 31° = 17$ — **Multiply each side by x.**

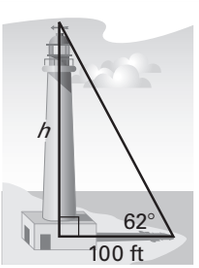
$x = \frac{17}{\tan 31°}$ — **Divide each side by $\tan 31°$.**

$x \approx \frac{17}{0.6009}$ — **Use a calculator to find $\tan 31°$.**

$x \approx 28.3$ — **Simplify.**

Example 3 ***Estimate height using tangent***

Lighthouse Find the height *h* of the lighthouse to the nearest foot.



$\tan 62° = \frac{\text{opp.}}{\text{adj.}}$ — **Write ratio for $\tan 62°$.**

$\tan 62° = \frac{h}{100}$ — **Substitute.**

$100 \cdot \tan 62° = h$ — **Multiply each side by 100.**

$188 \approx h$ — **Use a calculator and simplify.**

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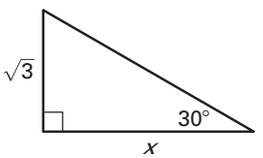
Your Notes

Example 4 ***Use a special right triangle to find a tangent***

Use a special right triangle to find the tangent of a 30° angle.

Solution

Step 1 **Choose** $\sqrt{3}$ as the length of the shorter leg to simplify calculations. Use the 30°-60°-90° Triangle Theorem to find the length of the longer leg.



longer leg = shorter leg • $\sqrt{3}$

$x = \sqrt{3} \cdot \sqrt{3} = 3$

Step 2 **Find** tan 30°.

$\tan 30° = \frac{\text{opp.}}{\text{adj.}}$ **Write ratio for tangent of 30°.**

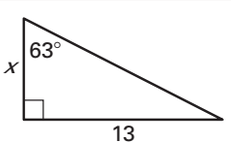
$\tan 30° = \frac{\sqrt{3}}{3}$ **Substitute.**

The tangent of any 30° angle is $\frac{\sqrt{3}}{3} \approx 0.5774$.

The tangents of all 30° angles are the same constant ratio. Any right triangle with a 30° angle can be used to determine this value.

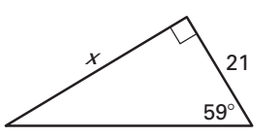
✔ ***Checkpoint*** **In Exercises 2 and 3, find the value of *x*. Round to the nearest tenth.**

2.



$x \approx 6.6$

3.



$x \approx 34.9$

4. In Example 4, suppose the length of the shorter leg is 1 instead of $\sqrt{3}$. Show that the tangent of 30° is still equal to $\frac{\sqrt{3}}{3}$.

longer leg = shorter leg • $\sqrt{3}$

$x = 1 \cdot \sqrt{3} = \sqrt{3}$

$\tan 30° = \frac{\text{opp.}}{\text{adj.}} = \frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

Homework

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