7.6 Apply the Sine and Cosine Ratios

Goal • Use the sine and cosine ratios.

Your Notes

VOCABULARY

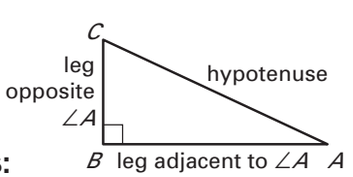
Sine, cosine Sine and cosine are trigonometric ratios for acute angles that involve the lengths of a leg and the hypotenuse of a right triangle.

Angle of elevation When looking up at an object, the angle your line of sight makes with a horizontal line is called the angle of elevation.

Angle of depression When looking down at an object, the angle your line of sight makes with a horizontal line is called the angle of depression.

SINE AND COSINE RATIOS

Let $\triangle ABC$ be a right triangle with acute $\angle A$. The sine of $\angle A$ and cosine of $\angle A$ (written sin A and cos A) are defined as follows:



Remember these abbreviations:
sine → sin
cosine → cos
hypotenuse → hyp

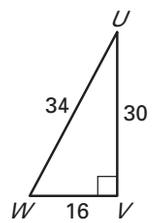
$$\sin A = \frac{\text{length of leg opposite } \angle A}{\text{length of hypotenuse}} = \frac{BC}{AC}$$

$$\cos A = \frac{\text{length of leg adjacent to } \angle A}{\text{length of hypotenuse}} = \frac{AB}{AC}$$

Your Notes

Example 1 *Find sine ratios*

Find sin *U* and sin *W*. Write each answer as a fraction and as a decimal rounded to four places.



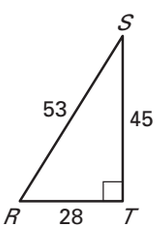
Solution

$$\sin U = \frac{\text{opp. } \angle U}{\text{hyp.}} = \frac{WV}{UW} = \frac{16}{34} = \frac{8}{17} \approx 0.4706$$

$$\sin W = \frac{\text{opp. } \angle W}{\text{hyp.}} = \frac{UV}{UW} = \frac{30}{34} = \frac{15}{17} \approx 0.8824$$

Example 2 *Find cosine ratios*

Find cos *S* and cos *R*. Write each answer as a fraction and as a decimal rounded to four places.



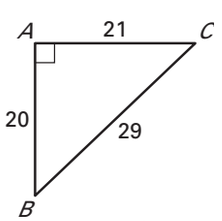
Solution

$$\cos S = \frac{\text{adj. to } \angle S}{\text{hyp.}} = \frac{ST}{SR} = \frac{45}{53} \approx 0.8491$$

$$\cos R = \frac{\text{adj. to } \angle R}{\text{hyp.}} = \frac{RT}{SR} = \frac{28}{53} \approx 0.5283$$

✔ ***Checkpoint*** **Find sin *B*, sin *C*, cos *B*, and cos *C*. Write each answer as a fraction and as a decimal rounded to four places.**

1.



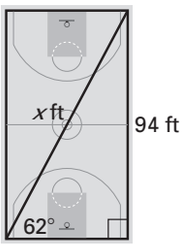
$\sin B = \frac{21}{29} \approx 0.7241$, $\sin C = \frac{20}{29} \approx 0.6897$,

$\cos B = \frac{20}{29} \approx 0.6897$, $\cos C = \frac{21}{29} \approx 0.7241$

Your Notes

Example 3 ***Use a trigonometric ratio to find a hypotenuse***

Basketball You walk from one corner of a basketball court to the opposite corner. Write and solve a proportion using a trigonometric ratio to approximate the distance of the walk.



Solution

$\sin 62° = \frac{\text{opp.}}{\text{hyp.}}$ **Write ratio for sine of 62°.**

$\sin 62° = \frac{94}{x}$ **Substitute.**

$x \cdot \sin 62° = 94$ **Multiply each side by** x.

$x = \frac{94}{\sin 62°}$ **Divide each side by** $\sin 62°$.

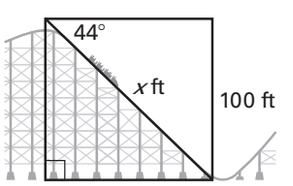
$x \approx \frac{94}{0.8829}$ **Use a calculator to find** $\sin 62°$.

$x \approx 106.5$ **Simplify.**

The distance of the walk is about 106.5 feet.

Example 4 ***Find a hypotenuse using an angle of depression***

Roller Coaster You are at the top of a roller coaster 100 feet above the ground. The angle of depression is 44°. About how far do you ride down the hill?



$\sin 44° = \frac{\text{opp.}}{\text{hyp.}}$ **Write ratio for sine of 44°.**

$\sin 44° = \frac{100}{x}$ **Substitute.**

$x \cdot \sin 44° = 100$ **Multiply each side by** x.

$x = \frac{100}{\sin 44°}$ **Divide each side by** $\sin 44°$.

$x \approx \frac{100}{0.6947}$ **Use a calculator to find** $\sin 44°$.

$x \approx 143.9$ **Simplify.**

You ride about 144 feet down the hill.

Your Notes

✔ ***Checkpoint*** **Complete the following exercises.**

2. In Example 3, use the cosine ratio to approximate the width of the basketball court.

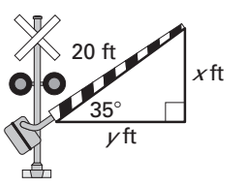
about 50 feet

3. Suppose the angle of depression in Example 4 is 72°. About how far would you ride down the hill?

about 105 feet

Example 5 ***Find leg lengths using an angle of elevation***

Railroad A railroad crossing arm that is 20 feet long is stuck with an angle of elevation of 35°. Find the lengths x and y.



Solution

Step 1 Find x.

$\sin 35° = \frac{\text{opp.}}{\text{hyp.}}$ — Write ratio for sine of 35°.

$\sin 35° = \frac{x}{20}$ — Substitute.

$20 \cdot \sin 35° = x$ — Multiply each side by 20.

$11.5 \approx x$ — Use a calculator to simplify.

Step 2 Find y.

$\cos 35° = \frac{\text{adj.}}{\text{hyp.}}$ — Write ratio for cosine of 35°.

$\cos 35° = \frac{y}{20}$ — Substitute.

$20 \cdot \cos 35° = y$ — Multiply each side by 20.

$16.4 \approx y$ — Use a calculator to simplify.

Your Notes

Example 6 ***Use a special right triangle to find a sin and cos***

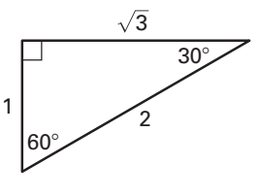
Use a special right triangle to find the sine and cosine of a 30° angle.

Solution

Use the 30°-60°-90° Triangle Theorem to draw a right triangle with side lengths of 1, $\sqrt{3}$, and 2. Then set up sine and cosine ratios for the 30° angle.

$$\sin 30° = \frac{\text{opp.}}{\text{hyp.}} = \frac{1}{2} = 0.5000$$

$$\cos 30° = \frac{\text{adj.}}{\text{hyp.}} = \frac{\sqrt{3}}{2} \approx 0.8660$$



Checkpoint **Complete the following exercises.**

4. In Example 5, suppose the angle of elevation is 40°. What are the new lengths x and y?

$x \approx 12.9$, $y \approx 15.3$

5. Use a special right triangle to find the sine and cosine of a 60° angle.

$\sin 60° \approx 0.8660$

$\cos 60° = 0.5000$

Homework