7.7 Solve Right Triangles

Goal • Use inverse tangent, sine, and cosine ratios.

Your Notes

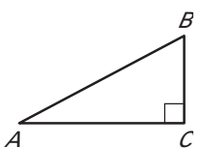
VOCABULARY

Solve a right triangle To solve a right triangle is to find the measures of all of its sides and angles.

The expression "$\tan^{-1} x$" is read as "the inverse tangent of x."

INVERSE TRIGONOMETRIC RATIOS

Let $\angle A$ be an acute angle.



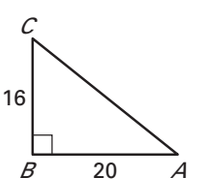
Inverse Tangent If $\tan A = x$, then $\tan^{-1} x = m\angle A$. $\tan^{-1} \frac{BC}{AC} = m\angle A$

Inverse Sine If $\sin A = y$, then $\sin^{-1} y = m\angle A$. $\sin^{-1} \frac{BC}{AB} = m\angle A$

Inverse Cosine If $\cos A = z$, then $\cos^{-1} z = m\angle A$. $\cos^{-1} \frac{AC}{AB} = m\angle A$

Example 1 ***Use an inverse tangent to find an angle measure***

Use a calculator to approximate the measure of $\angle A$ to the nearest tenth of a degree.



Because $\tan A = \frac{16}{20} = \frac{4}{5} =$ 0.8 ,

$\tan^{-1}$ 0.8 $= m\angle A$. Using a calculator,

$\tan^{-1}$ 0.8 $\approx$ 38.65980825 . . . .

So, the measure of $\angle A$ is approximately 38.7° .

✔ ***Checkpoint*** **Complete the following exercise.**

1. In Example 1, use a calculator and an inverse tangent to approximate $m\angle C$ to the nearest tenth of a degree.

$m\angle C \approx 51.3$

Your Notes

Example 2 ***Use an inverse sine and an inverse cosine***

Let $\angle A$ and $\angle B$ be acute angles in two right triangles. Use a calculator to approximate the measures of $\angle A$ and $\angle B$ to the nearest tenth of a degree.

a. $\sin A = 0.76$ **b.** $\cos B = 0.17$

Solution

a. $m\angle A =$ $\sin^{-1} 0.76$

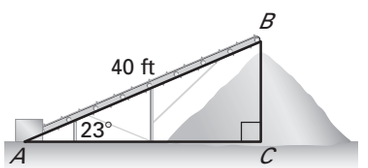
$\approx$ $49.5°$

b. $m\angle B =$ $\cos^{-1} 0.17$

$\approx$ $80.2°$

Example 3 ***Solve a right triangle***

Solve the right triangle. Round decimal answers to the nearest tenth.



Solution

Step 1 Find $m\angle B$ by using the Triangle Sum Theorem.

$180°$ $= 90° + 23° + m\angle B$

$67°$ $= m\angle B$

Step 2 Approximate BC using a sine ratio.

$\sin 23°$ $= \frac{BC}{40}$ **Write ratio for** $\sin 23°$ **.**

$40 \cdot \sin 23°$ $= BC$ **Multiply each side by** 40 **.**

$40 \cdot 0.3907$ $\approx BC$ **Approximate** $\sin 23°$ **.**

15.6 $\approx BC$ **Simplify and round answer.**

Step 3 Approximate AC using a cosine ratio.

$\cos 23°$ $= \frac{AC}{40}$ **Write ratio for** $\cos 23°$ **.**

$40 \cdot \cos 23°$ $= AC$ **Multiply each side by** 40 **.**

$40 \cdot 0.9205$ $\approx AC$ **Approximate** $\cos 23°$ **.**

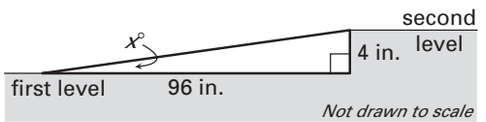
36.8 $\approx AC$ **Simplify and round answer.**

The angle measures are $23°$, $67°$, and $90°$. The side lengths are 40 feet, about 15.6 feet, and about 36.8 feet.

Your Notes

Example 4 ***Solve a real-world problem***

Model Train You are building a track for a model train. You want the track to incline from the first level to the second level, 4 inches higher, in 96 inches. Is the angle of elevation less than 3°?



Solution

Use the tangent and inverse tangent ratios to find the degree measure x of the incline.

$\tan x° = \frac{\text{opp.}}{\text{adj.}} = \frac{4}{96} \approx$ 0.0417

$x \approx \tan^{-1} 0.0417 \approx$ 2.4

The incline is about 2.4°, so it is less than 3°.

✔ ***Checkpoint*** **Complete the following exercises.**

2. Find $m\angle D$ to the nearest tenth of a degree if $\sin D = 0.48$.

$m\angle D \approx 28.7°$

3. Solve a right triangle that has a 50° angle and a 15 inch hypotenuse.

Angles: 90°, 50°, and 40°; Side lengths: 15 in., about 9.6 in., and about 11.5 in.

4. In Example 4, suppose another incline rises 8 inches in 120 inches. Is the incline less than 3°?

No, the incline is about 3.8°.

Homework