# 9.1 Translate Figures and Use Vectors

**Goal** • Use a vector to translate a figure.

***Your Notes***

**VOCABULARY**

**Image** An image is a new figure produced from the transformation of a figure.

**Preimage** A preimage is the original figure in the transformation of a figure.

**Isometry** An isometry is a transformation that preserves length and angle measure.

**Vector** A vector is a quantity that has both direction and magnitude, or size.

**Initial point** The initial point of a vector is the starting point of the vector.

**Terminal point** The terminal point of a vector is the ending point of the vector.

**Horizontal component** The horizontal component describes the left and right direction of a vector.

**Vertical component** The vertical component describes the up and down direction of a vector.

**Component form** The component form of a vector combines the horizontal and vertical components.

**Your Notes**

You can use *prime notation* to name an image. For example, if the preimage is $\triangle ABC$, then its image is $\triangle A'B'C'$, read as "*triangle A prime, B prime, C prime.*"

## Example 1 *Translate a figure in the coordinate plane*

**Graph quadrilateral *ABCD* with vertices $A(-2, 6)$, $B(2, 4)$, $C(2, 1)$, and $D(-2, 3)$. Find the image of each vertex after the translation $(x, y) \rightarrow (x + 3, y - 3)$. Then graph the image using prime notation.**

### Solution

First, draw *ABCD*. Find the translation of each vertex by __adding__ 3 to its *x*-coordinate and __subtracting__ 3 from its *y*-coordinate. Then graph the image.

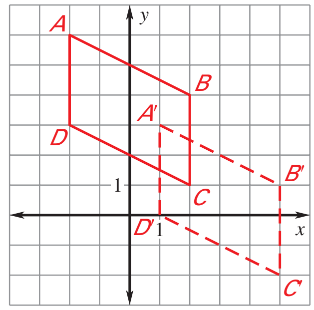
$(x, y) \rightarrow (x + 3, y - 3)$

$A(-2, 6) \rightarrow A'($__1, 3__$)$

$B(2, 4) \rightarrow B'($__5, 1__$)$

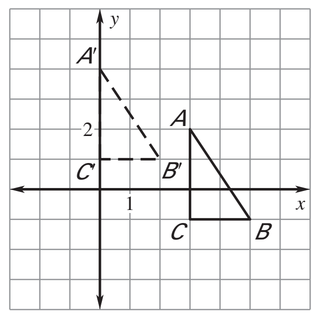
$C(2, 1) \rightarrow C'($__5, −2__$)$

$D(-2, 3) \rightarrow D'($__1, 0__$)$




## Example 2 *Write a translation rule and verify congruence*

**Write a rule for the translation of $\triangle ABC$ to $\triangle A'B'C'$. Then verify that the transformation is an isometry.**



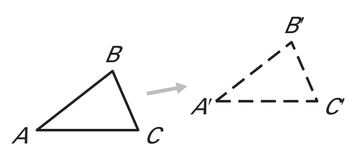

### Solution

To go from *A* to *A'*, move 3 units __left__ and 2 units __up__. So, a rule for the translation is $(x, y) \rightarrow ($__$x - 3, y + 2$__$)$.

Use the SAS Congruence Postulate. Notice that $CB = C'B' =$ __2__, and $AC = A'C' =$ __3__. The slopes of $\overline{CB}$ and $\overline{C'B'}$ are __0__, and the slopes of $\overline{CA}$ and $\overline{C'A'}$ are __undefined__, so the sides are __perpendicular__. Therefore, $\angle C$ and $\angle C'$ are __congruent right angles__. So, $\triangle ABC$ __$\cong$__ $\triangle A'B'C'$. The translation is an isometry.

**THEOREM 9.1: TRANSLATION THEOREM**

A translation is an isometry.




$\triangle ABC \cong \triangle A'B'C'$

*Your Notes*

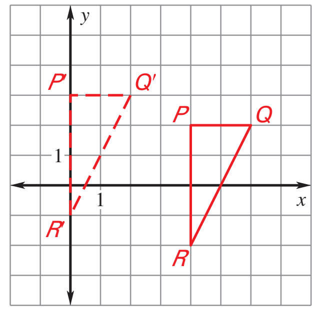
**✔ *Checkpoint*** **Complete the following exercises.**

**1.** Draw $\triangle PQR$ with vertices $P(4, 2)$, $Q(6, 2)$, and $R(4, -2)$. Find the image of each vertex after the translation $(x, y) \rightarrow (x - 4, y + 1)$. Graph the image using prime notation.

$P'(0, 3)$

$Q'(2, 3)$

$R'(0, -1)$



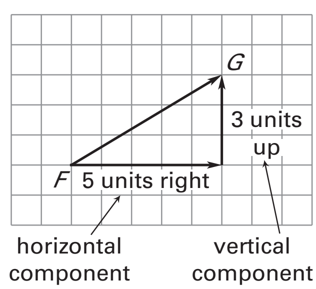

**2.** In Example 2, write a rule to translate $\triangle A'B'C'$ back to $\triangle ABC$.

$(x, y) \rightarrow (x + 3, y - 2)$

## VECTORS

The diagram shows a vector named $\overrightarrow{FG}$, read as "vector *FG*."

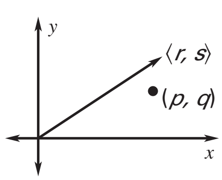
The initial point, or starting point, of the vector is __F__.




The terminal point, or ending point, of the vector is __G__.

The component form of a vector combines the horizontal and vertical components. So, the component form of $\overrightarrow{FG}$ is __$\langle 5, 3 \rangle$__.

Use brackets to write the component form of the vector $\langle r, s \rangle$. Use parentheses to write the coordinates of the point $(p, q)$.

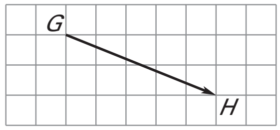


***Your Notes***

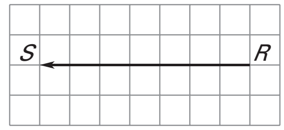
## Example 3 *Identify vector components*

**Name the vector and write its component form.**

**a.**




**b.**



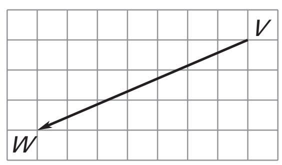

**Solution**

**a.** The vector is $\overrightarrow{GH}$. From initial point G to terminal point H, you move 5 units right and 2 units down. So, the component form is ⟨5, −2⟩.

**b.** The vector is $\overrightarrow{RS}$. From initial point R to terminal point S, you move 7 units left and 0 units vertically. So, the component form is ⟨−7, 0⟩.

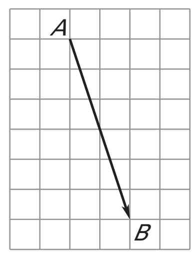
✔ ***Checkpoint*** **Name the vector and write its component form.**

**3.**




$\overrightarrow{VW}$; ⟨−7, −3⟩

**4.**




$\overrightarrow{AB}$; ⟨2, −6⟩

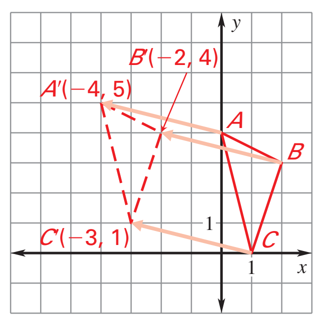
## Example 4 *Use a vector to translate a figure*

**The vertices of △*ABC* are *A*(0, 4), *B*(2, 3), and *C*(1, 0). Translate △*ABC* using the vector ⟨−4, 1⟩.**

Notice that the vector can have different initial points. The vector describes only the direction and magnitude of the translation.

**Solution**

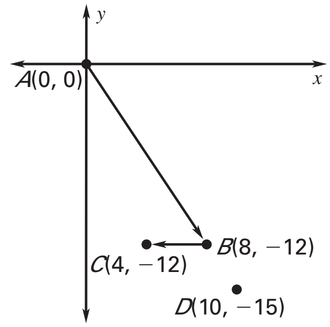
First, graph △*ABC*. Use ⟨−4, 1⟩ to move each vertex 4 units to the left and 1 unit up. Label the image vertices. Draw △*A′B′C′*. Notice that the vectors drawn from preimage to image vertices are parallel.



*Your Notes*

## Example 5 *Solve a multi-step problem*

**Construction** A car heads out from point $A$ toward point $D$. The car encounters construction at $B$, 8 miles east and 12 miles south of its starting point. The detour route leads the car to point $C$, as shown.




**a.** Write the component form of $\overrightarrow{AB}$.

**b.** Write the component form of $\overrightarrow{BC}$.

**c.** Write the component form of the vector that describes the straight line path from the car's current position $C$ to its intended destination $D$.

**a.** The component form of the vector from $A(0, 0)$ to $B(8, -12)$ is

$\overrightarrow{AB} = \langle 8 - 0, -12 - 0 \rangle = \langle 8, -12 \rangle$.

**b.** The component form of the vector from $B(8, -12)$ to $C(4, -12)$ is

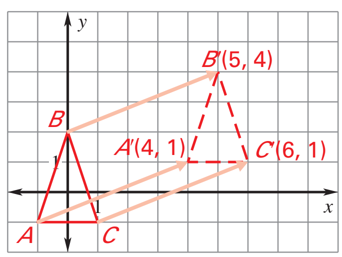
$\overrightarrow{BC} = \langle 4 - 8, -12 - (-12) \rangle = \langle -4, 0 \rangle$.

**c.** The car is currently at point $C$ and needs to travel to $D$. The component form of the vector from $C(4, -12)$ to $D(10, -15)$ is

$\overrightarrow{CD} = \langle 10 - 4, -15 - (-12) \rangle = \langle 6, -3 \rangle$.

***Checkpoint*** **Complete the following exercises.**

**5.** The vertices of $\triangle ABC$ are $A(-1, -1)$, $B(0, 2)$, and $C(1, -1)$. Translate $\triangle ABC$ using the vector $\langle 5, 2 \rangle$.




**6.** In Example 5, suppose there is no construction. Write the component form of the vector that describes the straight path from the car's starting point $A$ to its final destination $D$.

$\langle 10, -15 \rangle$

***Homework***