9.2 Use Properties of Matrices

Goal • Perform translations using matrix operations.

Your Notes

VOCABULARY

Matrix A matrix is a rectangular arrangement of numbers in rows and columns.

Element Each number in a matrix is called an element.

Dimensions The dimensions of a matrix are the numbers of rows and columns.

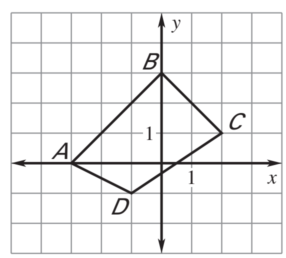
An element of a matrix may also be called an *entry*.

Example 1 ***Represent figures using matrices***

Write a matrix to represent the point or polygon.

a. Point *A*

b. Quadrilateral *ABCD*



Solution

a. Point matrix for *A*

$$\begin{bmatrix} -3 \\ 0 \end{bmatrix} \begin{array}{l} \leftarrow x\text{-coordinate} \\ \leftarrow y\text{-coordinate} \end{array}$$

b. Polygon matrix for *ABCD*

$$\begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} -3 & 0 & 2 & -1 \\ 0 & 3 & 1 & -1 \end{bmatrix} \end{array} \begin{array}{l} \leftarrow x\text{-coordinates} \\ \leftarrow y\text{-coordinates} \end{array}$$

The columns in a polygon matrix follow the consecutive order of the vertices of the polygon.

✔ ***Checkpoint*** **Complete the following exercise.**

1. Write a matrix to represent $\triangle RST$ with vertices $R(-5, -4)$, $S(-1, 2)$, and $T(3, 1)$.

$$\begin{array}{c} \begin{array}{ccc} R & S & T \end{array} \\ \begin{bmatrix} -5 & -1 & 3 \\ -4 & 2 & 1 \end{bmatrix} \end{array}$$

Your Notes

Example 2 *Add and subtract matrices*

a. $\begin{bmatrix} 4 & -2 \\ 2 & -3 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 5 & -6 \end{bmatrix} = \begin{bmatrix} 4+1 & -2+2 \\ 2+5 & -3+(-6) \end{bmatrix}$

$= \begin{bmatrix} 5 & 0 \\ 7 & -9 \end{bmatrix}$

b. $\begin{bmatrix} 7 & 4 & 5 \\ 1 & -2 & 8 \end{bmatrix} - \begin{bmatrix} 3 & -6 & 5 \\ 0 & 7 & 1 \end{bmatrix}$

$= \begin{bmatrix} 7-3 & 4-(-6) & 5-5 \\ 1-0 & -2-7 & 8-1 \end{bmatrix}$

$= \begin{bmatrix} 4 & 10 & 0 \\ 1 & -9 & 7 \end{bmatrix}$

Example 3 *Represent a translation using matrices*

The matrix $\begin{bmatrix} 2 & 3 & 4 \\ -3 & 2 & 0 \end{bmatrix}$ represents $\triangle ABC$. Find the image matrix that represents the translation of $\triangle ABC$ 4 units left and 1 unit down. Then graph $\triangle ABC$ and its image.

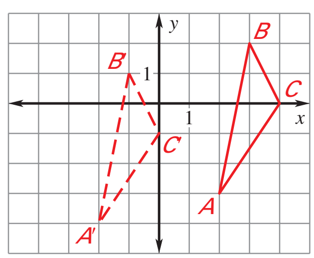
In order to add two matrices, they must have the same dimensions, so the translation matrix here must have three columns like the polygon matrix.

Solution

The translation matrix is $\begin{bmatrix} -4 & -4 & -4 \\ -1 & -1 & -1 \end{bmatrix}$.

Add this to the polygon matrix for the preimage to find the image matrix.

$$\underset{\text{Translation matrix}}{\begin{bmatrix} -4 & -4 & -4 \\ -1 & -1 & -1 \end{bmatrix}} + \underset{\text{Polygon matrix}}{\begin{array}{ccc} A & B & C \\ \end{array} \atop \begin{bmatrix} 2 & 3 & 4 \\ -3 & 2 & 0 \end{bmatrix}} = \underset{\text{Image matrix}}{\begin{array}{ccc} A' & B' & C' \\ \end{array} \atop \begin{bmatrix} -2 & -1 & 0 \\ -4 & 1 & -1 \end{bmatrix}}$$



Your Notes

Example 4 *Multiply matrices*

Multiply $\begin{bmatrix} 0 & 4 \\ 5 & 2 \end{bmatrix}\begin{bmatrix} -4 & 1 \\ 8 & -3 \end{bmatrix}$.

Solution

The matrices are both 2 × 2, so their product is defined. Use the following steps to find the elements of the product matrix.

Step 1 **Multiply** the numbers in the <u>first row</u> of the first matrix by the numbers in the <u>first column</u> of the second matrix. Put the result in the first row, first column of the product matrix.

$$\begin{bmatrix} 0 & 4 \\ 5 & 2 \end{bmatrix}\begin{bmatrix} -4 & 1 \\ 8 & -3 \end{bmatrix} = \begin{bmatrix} \underline{0(-4) + 4(8)} & ? \\ ? & ? \end{bmatrix}$$

Step 2 **Multiply** the numbers in the <u>first row</u> of the first matrix by the numbers in the <u>second column</u> of the second matrix. Put the result in the first row, second column of the product matrix.

$$\begin{bmatrix} 0 & 4 \\ 5 & 2 \end{bmatrix}\begin{bmatrix} -4 & 1 \\ 8 & -3 \end{bmatrix} = \begin{bmatrix} \underline{0(-4) + 4(8)} & \underline{0(1) + 4(-3)} \\ ? & ? \end{bmatrix}$$

Step 3 **Multiply** the numbers in the <u>second row</u> of the first matrix by the numbers in the <u>first column</u> of the second matrix. Put the result in the second row, first column of the product matrix.

$$\begin{bmatrix} 0 & 4 \\ 5 & 2 \end{bmatrix}\begin{bmatrix} -4 & 1 \\ 8 & -3 \end{bmatrix} = \begin{bmatrix} \underline{0(-4) + 4(8)} & \underline{0(1) + 4(-3)} \\ \underline{5(-4) + 2(8)} & ? \end{bmatrix}$$

Step 4 **Multiply** the numbers in the <u>second row</u> of the first matrix by the numbers in the <u>second column</u> of the second matrix. Put the result in the second row, second column of the product matrix.

$$\begin{bmatrix} 0 & 4 \\ 5 & 2 \end{bmatrix}\begin{bmatrix} -4 & 1 \\ 8 & -3 \end{bmatrix} = \begin{bmatrix} \underline{0(-4) + 4(8)} & \underline{0(1) - 4(-3)} \\ \underline{5(-4) + 2(8)} & \underline{5(1) + 2(-3)} \end{bmatrix}$$

Step 5 **Simplify** the product matrix.

$$\begin{bmatrix} 0 & 4 \\ 5 & 2 \end{bmatrix}\begin{bmatrix} -4 & 1 \\ 8 & -3 \end{bmatrix} = \begin{bmatrix} \underline{32} & \underline{-12} \\ \underline{-4} & \underline{-1} \end{bmatrix}$$

Your Notes

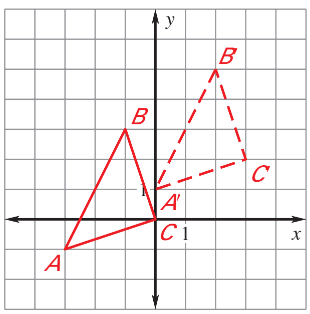
Checkpoint Complete the following exercises.

2. Subtract $\begin{bmatrix} 3 & -5 \\ 8 & -4 \end{bmatrix} - \begin{bmatrix} 9 & 7 \\ -3 & 1 \end{bmatrix}$.

$\begin{bmatrix} -6 & -12 \\ 11 & -5 \end{bmatrix}$

3. The matrix $\begin{bmatrix} -3 & -1 & 0 \\ -1 & 3 & 0 \end{bmatrix}$ represents $\triangle ABC$. Find the image matrix that represents the translation of $\triangle ABC$ 3 units right and 2 units up. Then graph $\triangle ABC$ and its image.

$\begin{bmatrix} 0 & 2 & 3 \\ 1 & 5 & 2 \end{bmatrix}$



4. Multiply $\begin{bmatrix} 6 & 3 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$.

$\begin{bmatrix} 6 & -3 \\ 2 & 1 \end{bmatrix}$

Your Notes

Example 5 ***Solve a real-world problem***

Hockey A men's hockey team m needs 7 sticks, 30 pucks, and 4 helmets. A women's team w needs 5 sticks, 25 pucks, and 5 helmets. A hockey stick costs $30, a puck costs $4, and a helmet costs $50. Use matrix multiplication to find the total cost of equipment for each team.

Solution

Write equipment needs and costs per item in matrix form. You will use matrix multiplication, so form the matrices so that the number of columns of the equipment matrix matches the number of rows of the cost per item matrix.

You could solve this problem arithmetically, multiplying the number of sticks by the price of sticks, and so on, then adding the costs for each team.

Equipment • **Cost** = **Total Cost**

$$\begin{array}{c c} & \begin{array}{ccc} \text{Sticks} & \text{Pucks} & \text{Helmets} \end{array} \\ \begin{array}{c} m \\ w \end{array} & \begin{bmatrix} 7 & 30 & 4 \\ 5 & 25 & 5 \end{bmatrix} \end{array} \cdot \begin{array}{c c} & \text{Dollars} \\ \begin{array}{r} \text{Stick} \\ \text{Puck} \\ \text{Helmet} \end{array} & \begin{bmatrix} 30 \\ 4 \\ 50 \end{bmatrix} \end{array} = \begin{array}{c c} & \text{Dollars} \\ \begin{array}{c} m \\ w \end{array} & \begin{bmatrix} ? \\ ? \end{bmatrix} \end{array}$$

You can find the total cost of equipment for each team by multiplying the equipment matrix by the cost per item matrix. The equipment matrix is 2 × 3 and the cost per item matrix is 3 × 1, so their product is a 2 × 1 matrix.

$$\begin{bmatrix} 7 & 30 & 4 \\ 5 & 25 & 5 \end{bmatrix} \begin{bmatrix} 30 \\ 4 \\ 50 \end{bmatrix} = \begin{bmatrix} 7(30) + 30(4) + 4(50) \\ 5(30) + 25(4) + 5(50) \end{bmatrix} = \begin{bmatrix} 530 \\ 500 \end{bmatrix}$$

The total cost of equipment for the men's team is $530, and the total cost for the women's team is $500.

✔ ***Checkpoint*** **Complete the following exercise.**

5. In Example 5, find the total costs if a stick costs $50, a puck costs $2, and a helmet costs $70.

Men's team: $690, Women's team: $650

Homework