9.3 Perform Reflections

Goal • Reflect a figure in any given line.

Your Notes

VOCABULARY

Line of reflection In a reflection, the mirror line is called the line of reflection.

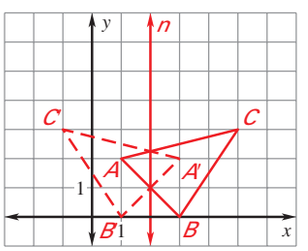
Example 1 ***Graph reflections in horizontal and vertical lines***

The vertices of $\triangle ABC$ are $A(1, 2)$, $B(3, 0)$, and $C(5, 3)$. Graph the reflection of $\triangle ABC$ described.

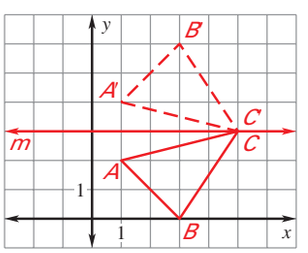
a. In the line n: $x = 2$ **b.** In the line m: $y = 3$

Solution

a. Point A is 1 unit left of n, so its reflection A' is 1 unit right of n at (3 , 2). Also, B' is 1 unit left of n at (1 , 0), and C' is 3 units left of n at (−1 , 3).

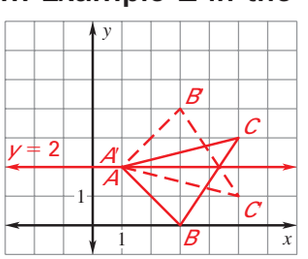


b. Point A is 1 unit below m, so A' is 1 unit above m at (1 , 4). Also, B' is 3 units above m at (3 , 6). Because point C is on line m, you know that $C =$ C' .



✔ ***Checkpoint*** **Complete the following exercise.**

1. Graph the reflection of $\triangle ABC$ from Example 1 in the line $y = 2$.



Your Notes

Example 2 ***Graph a reflection in $y = x$***

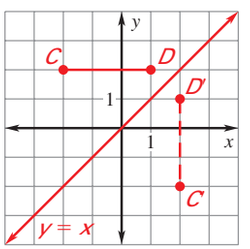
The endpoints of $\overline{CD}$ are $C(-2, 2)$ and $D(1, 2)$. Reflect the segment in the line $y = x$. Graph the segment and its image.

The product of the slopes of perpendicular lines is -1.

Solution

The slope of $y = x$ is __1__. The segment from C to its image, $\overline{CC'}$, is __perpendicular__ to the line of reflection $y = x$, so the slope of $\overline{CC'}$ will be __-1__ (because $1(-1) =$ __-1__). From C, move __2__ units right and __2__ units down to $y = x$. From that point, move __2__ units right and __2__ units down to locate C'(__2__, __-2__).

The slope of $\overline{DD'}$ will also be __-1__. From D, move __0.5__ units right and __0.5__ units down to $y = x$. Then move __0.5__ units right and __0.5__ units down to locate D'(__2__, __1__).



COORDINATE RULES FOR REFLECTIONS

- If (a, b) is reflected in the x-axis, its image is the point (__a__, __$-b$__).
- If (a, b) is reflected in the y-axis, its image is the point (__$-a$__, __b__).
- If (a, b) is reflected in the line $y = x$, its image is the point (__b__, __a__).
- If (a, b) is reflected in the line $y = -x$, its image is the point (__$-b$__, __$-a$__).

Your Notes

Example 3 ***Graph a reflection in $y = -x$***

Reflect $\overline{CD}$ from Example 2 in the line $y = -x$. Graph $\overline{CD}$ and its image.

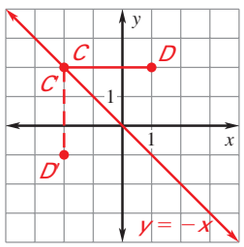
Solution

Use the coordinate rule for reflecting in the line $y = -x$.

$(a, b) \rightarrow (-b, -a)$

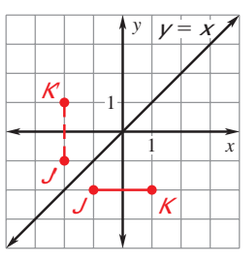
$C(-2, 2) \rightarrow C'(\underline{-2}, \underline{2})$

$D(1, 2) \rightarrow D'(\underline{-2}, \underline{-1})$

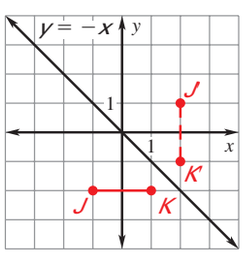


✔ ***Checkpoint*** **The endpoints of $\overline{JK}$ are $J(-1, -2)$ and $K(1, -2)$. Reflect the segment in the given line. Graph the segment and its image.**

2. $y = x$

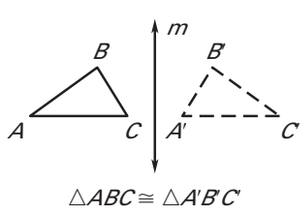


3. $y = -x$



THEOREM 9.2: REFLECTION THEOREM

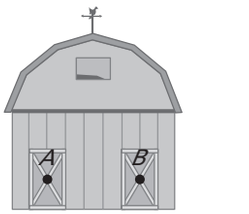
A reflection is an isometry.



Your Notes

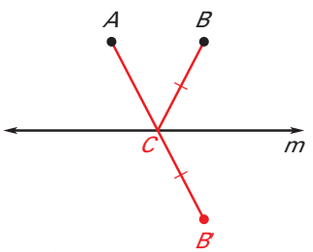
Example 4 *Find a minimum distance*

Tools Workers are retrieving tools that they need for a project. One will enter the building at point *A* and the other at point *B*. Where should they park on driveway *m* to minimize the distance they will walk?



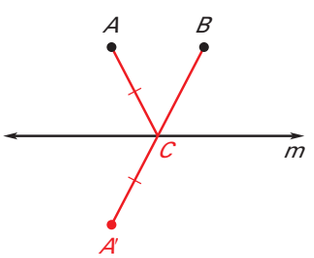
Solution

Reflect *B* in line *m* to obtain *B′*. Then draw $\overline{AB'}$. Label the intersection of $\overline{AB'}$ and *m* as *C*. Because *AB′* is the shortest distance between *A* and *B′* and *BC* = *B′C*, park at point *C* to minimize the combined distance, *AC* + *BC*, they have to walk.



Checkpoint **Complete the following exercise.**

4. In Example 4, reflect *A* in line *m*. What do you notice?

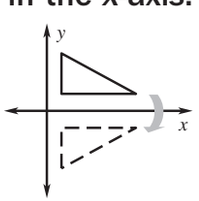


You obtain the same point *C* at which to park.

REFLECTION MATRICES

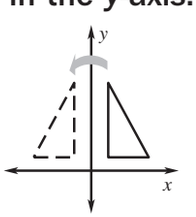
Reflection in the *x*-axis.

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$



Reflection in the *y*-axis.

$$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$



Your Notes

Example 5 ***Use matrix multiplication to reflect a polygon***

The vertices of $\triangle DEF$ are $D(1, 2)$, $E(2, 3)$, and $F(4, 1)$. Find the reflection of $\triangle DEF$ in the y-axis using matrix multiplication. Graph $\triangle DEF$ and its image.

Solution

Step 1 **Multiply** the polygon matrix by the matrix for a reflection in the y-axis.

$$\begin{matrix} & & D & E & F \end{matrix}$$

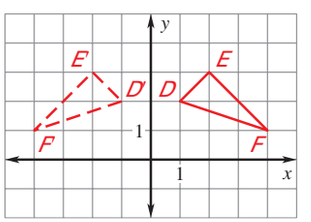
Reflection matrix $\rightarrow \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 4 \\ 2 & 3 & 1 \end{bmatrix} \leftarrow$ **Polygon matrix**

$$= \begin{bmatrix} -1(1) + 0(2) & -1(2) + 0(3) & -1(4) + 0(1) \\ 0(1) + 1(2) & 0(2) + 1(3) & 0(4) + 1(1) \end{bmatrix}$$

$$\begin{matrix} & D' & E' & F' \end{matrix}$$

$$= \begin{bmatrix} -1 & -2 & -4 \\ 2 & 3 & 1 \end{bmatrix}$$

Step 2 **Graph $\triangle DEF$ and $\triangle D'E'F'$.**



Checkpoint **Complete the following exercise.**

5. The vertices of $\triangle QRS$ are $Q(-1, 4)$, $R(0, 1)$, and $S(2, 3)$. Find the reflection of $\triangle QRS$ in the x-axis using matrix multiplication.

$Q'(-1, -4)$, $R'(0, -1)$, $S'(2, -3)$

Homework