9.4 Perform Rotations

Goal • Rotate figures about a point.

Your Notes

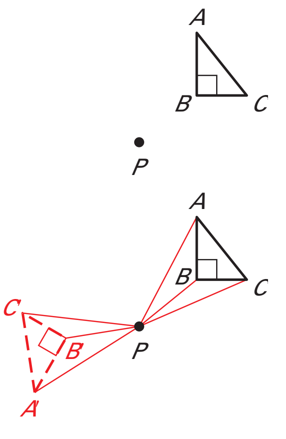
VOCABULARY

Center of rotation In a rotation, a figure is turned about a fixed point called the center of rotation.

Angle of rotation In a rotation, rays drawn from the center of rotation to a point and its image form the angle of rotation.

Example 1 ***Draw a rotation***

Draw a 150° rotation of $\triangle ABC$ about P.



Solution

Step 1 **Draw** a segment from A to P.

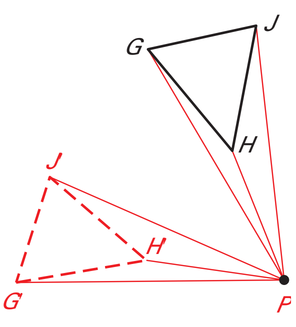
Step 2 **Draw** a ray to form a 150° angle with $\overline{PA}$.

Step 3 **Draw** A' so that $PA' = PA$.

Step 4 **Repeat** Steps 1–3 for each vertex. Draw $\triangle A'B'C'$.

Checkpoint **Complete the following exercise.**

1. Draw a 60° rotation of $\triangle GHJ$ about P.

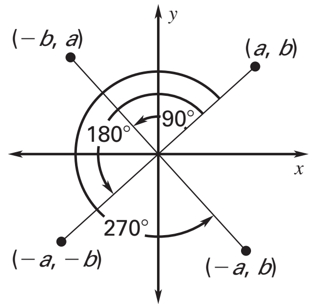


Your Notes

COORDINATE RULES FOR ROTATIONS ABOUT THE ORIGIN

When a point (a, b) is rotated counterclockwise about the origin, the following are true:

1. For a rotation of 90°, $(a, b) \rightarrow (\underline{-b}, \underline{a})$.
2. For a rotation of 180°, $(a, b) \rightarrow (\underline{-a}, \underline{-b})$.
3. For a rotation of 270°, $(a, b) \rightarrow (\underline{b}, \underline{-a})$.



Example 2 ***Rotate a figure using the coordinate rules***

Graph quadrilateral *KLMN* with vertices *K*(3, 2), *L*(4, 2), *M*(4, −3), and *N*(2, −1). Then rotate the quadrilateral 270° about the origin.

Solution

Graph *KLMN*. Use the coordinate rule for a 270° rotation to find the images of the vertices.

$(a, b) \rightarrow (b, -a)$

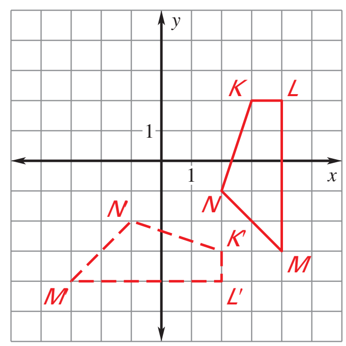
$K(3, 2) \rightarrow K'(\underline{2}, \underline{-3})$

$L(4, 2) \rightarrow L'(\underline{2}, \underline{-4})$

$M(4, -3) \rightarrow M'(\underline{-3}, \underline{-4})$

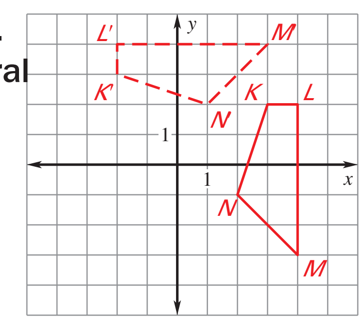
$N(2, -1) \rightarrow N'(\underline{-1}, \underline{-2})$

Graph the image $K'L'M'N'$.



Checkpoint **Complete the following exercise.**

2. Graph *KLMN* in Example 2. Then rotate the quadrilateral 90° about the origin.



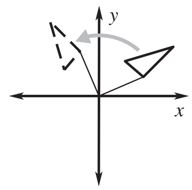
Your Notes

> Notice that a 360° rotation returns the figure to its original position. The matrix that represents this rotation is called the *identity matrix*.

ROTATION MATRICES (COUNTERCLOCKWISE)

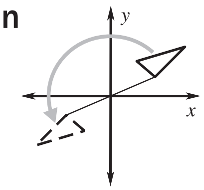
90° rotation

$$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$



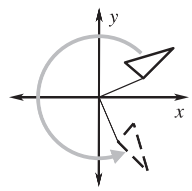
180° rotation

$$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$



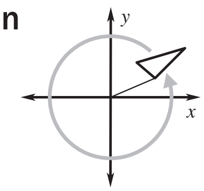
270° rotation

$$\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$



360° rotation

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$



Example 3 *Use matrices to rotate a figure*

Trapezoid *DEFG* has vertices *D*(−1, 3), *E*(1, 3), *F*(2, 1), and *G*(1, 0). Find the image matrix for a 180° rotation of *DEFG* about the origin. Graph *DEFG* and its image.

Solution

Step 1 **Write** the polygon matrix:

$$\begin{array}{c} \begin{matrix} \;D & \;\;E & \;\;F & \;\;G \end{matrix} \\ \begin{bmatrix} -1 & 1 & 2 & 1 \\ 3 & 3 & 1 & 0 \end{bmatrix} \end{array}$$

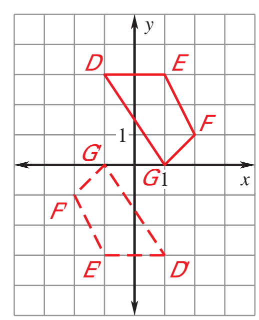
> Because matrix multiplication is not commutative, always write the rotation matrix first, then the polygon matrix.

Step 2 **Multiply** by the matrix for a 180° rotation.

$$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{array}{c} \begin{matrix} \;D & \;\;E & \;\;F & \;\;G \end{matrix} \\ \begin{bmatrix} -1 & 1 & 2 & 1 \\ 3 & 3 & 1 & 0 \end{bmatrix} \end{array} = \begin{array}{c} \begin{matrix} \;D' & \;E' & \;F' & \;G' \end{matrix} \\ \begin{bmatrix} 1 & -1 & -2 & -1 \\ -3 & -3 & 1 & 0 \end{bmatrix} \end{array}$$

Rotation matrix **Polygon matrix** **Image matrix**

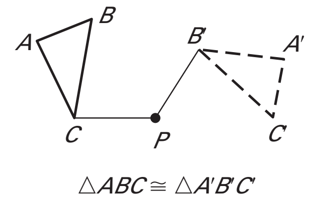
Step 3 **Graph** the preimage *DEFG*. Graph the image *D′E′F′G′*.



Your Notes

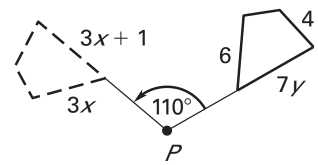
THEOREM 9.3: ROTATION THEOREM

A rotation is an isometry.



Example 4 *Find side lengths in a rotation*

The quadrilateral is rotated about P. Find the value of y.



Solution

By Theorem 9.3, the rotation is an isometry, so corresponding side lengths are equal. Then $3x = 6$, so $x = 2$. Now set up an equation to solve for y.

$7y = 3x + 1$	**Corresponding lengths in an isometry are equal.**
$7y = 3(2) + 1$	**Substitute 2 for x.**
$y = 1$	**Solve for y.**

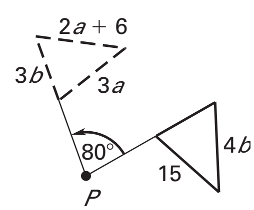
Checkpoint **Complete the following exercises.**

3. Use the quadrilateral in Example 3. Find the image matrix after a 270° rotation about the origin.

$$\begin{array}{cccc} D' & E' & F' & G' \end{array}$$
$$\begin{bmatrix} 3 & 3 & 1 & 0 \\ 1 & -1 & -2 & -1 \end{bmatrix}$$

4. The triangle is rotated about P. Find the value of b.

$b = 4$



Homework