9.5 Apply Compositions of Transformations

Goal • Perform combinations of two or more transformations.

Your Notes

VOCABULARY

Glide reflection A glide reflection is a transformation in which every point P is mapped to a point P'' by the following steps:
(1) A translation maps P onto P'.
(2) A reflection in a line k parallel to the direction of the translation maps P' to P''.

Composition of transformations When two or more transformations are combined to form a single transformation, the result is a composition of transformations.

Example 1 ***Find the image of a glide reflection***

The vertices of $\triangle ABC$ are $A(2, 1)$, $B(5, 3)$, and $C(6, 2)$. Find the image of $\triangle ABC$ after the glide reflection.

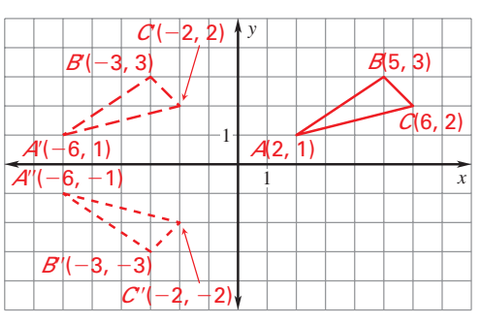
Translation: $(x, y) \rightarrow (x - 8, y)$

Reflection: in the x-axis

The line of reflection must be parallel to the direction of the translation to be a glide reflection.

Solution

Begin by graphing $\triangle ABC$. Then graph $\triangle A'B'C'$ after a translation 8 units left. Finally, graph $\triangle A''B''C''$ after a reflection in the x-axis.



Your Notes

THEOREM 9.4: COMPOSITION THEOREM

The composition of two (or more) isometries is an isometry.

Example 2 ***Find the image of a composition***

The endpoints of $\overline{CD}$ are $C(-2, 6)$ and $D(-1, 3)$. Graph the image of $\overline{CD}$ after the composition.

Unless you are told otherwise, do the transformations in the order given.

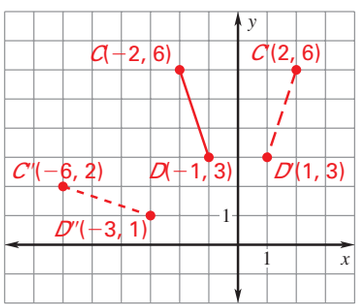
Reflection: in the y-axis
Rotation: 90° about the origin

Solution

Step 1 Graph $\overline{CD}$.

Step 2 Reflect $\overline{CD}$ in the y-axis. $\overline{C'D'}$ has endpoints $C'(\_2\_, \_6\_)$ and $D'(\_1\_, \_3\_)$.

Step 3 Rotate $\overline{C'D'}$ 90° about the origin. $\overline{C''D''}$ has endpoints $C''(\_-6\_, \_2\_)$ and $D''(\_-3\_, \_1\_)$.

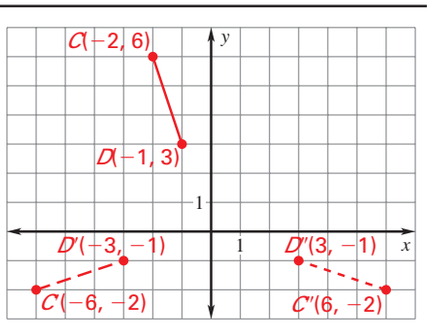


Checkpoint **Complete the following exercises.**

1. Suppose $\triangle ABC$ in Example 1 is translated 5 units down, then reflected in the y-axis. What are the coordinates of the vertices of the image?

$A''(-2, -4)$, $B''(-5, -2)$, $C''(-6, -3)$

2. Graph $\overline{CD}$ from Example 2. Do the rotation first, followed by the reflection.



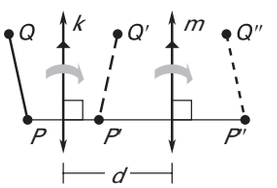
Your Notes

THEOREM 9.5: REFLECTIONS IN PARALLEL LINES THEOREM

If lines k and m are parallel, then a reflection in line k followed by a reflection in line m is the same as a translation.

If P'' is the image of P, then:

1. $\overline{PP''}$ is perpendicular to k and m, and
2. $PP'' = 2d$, where d is the distance between k and m.



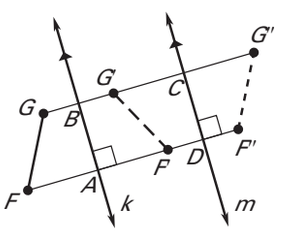
Example 3 ***Use Theorem 9.5***

In the diagram, a reflection in line k maps $\overline{GF}$ to $\overline{G'F'}$. A reflection in line m maps $\overline{G'F'}$ to $\overline{G''F''}$. Also, $FA = 6$ and $DF'' = 3$.

a. Name any segments congruent to each segment: $\overline{GF}$, $\overline{FA}$, and $\overline{GB}$.

b. Does $AD = BC$? *Explain*.

c. What is the length of $\overline{GG''}$?



a. $\overline{GF} \cong$ $\overline{G'F'}$, and $\overline{GF} \cong$ $\overline{G''F''}$. $\overline{FA} \cong$ $\overline{F'A}$. $\overline{GB} \cong$ $\overline{G'B}$.

b. Yes, AD = BC because $\overline{GG''}$ and $\overline{FF''}$ are perpendicular to both k and m, so $\overline{BC}$ and $\overline{AD}$ are opposite sides of a rectangle.

c. By the properties of reflections, $F'A =$ 6 and $F'D =$ 3. Theorem 9.5 implies that $GG'' = FF'' =$ 2 $\cdot$ AD, so the length of $\overline{GG''}$ is 2 (6 + 3), or 18 units.

✔ ***Checkpoint*** **Complete the following exercise.**

3. In Example 3, suppose you are given that $BC = 10$ and $G'F' = 6$. What is the perimeter of quadrilateral $GG''F''F$?

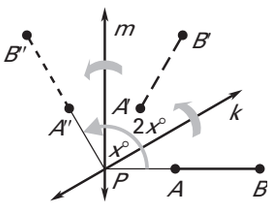
52 units

Your Notes

THEOREM 9.6: REFLECTIONS IN INTERSECTING LINES THEOREM

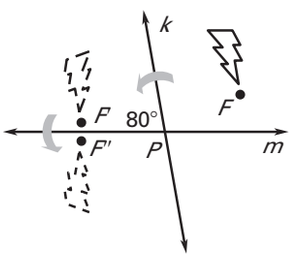
If lines k and m intersect at point P, then a reflection in k followed by a reflection in m is the same as a rotation about P.

The angle of rotation is $2x°$, where $x°$ is the measure of the acute or right angle formed by k and m.



Example 4 ***Use Theorem 9.6***

In the diagram, the figure is reflected in line k. The image is then reflected in line m. Describe a single transformation that maps F to F''.



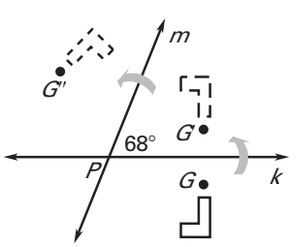
Solution

The measure of the acute angle formed between lines k and m is 80°. So, by Theorem 9.6, a single transformation that maps F to F'' is a 160° rotation about point P.

You can check that this is correct by tracing lines k and m and point F, then rotating the point 160°.

✔ ***Checkpoint*** **Complete the following exercise.**

4. In the diagram below, the preimage is reflected in line k, then in line m. Describe a single transformation that maps G to G''.



136° rotation about point P

Homework