

# 9.7

## Identify and Perform Dilations

- Goal** • Use drawing tools and matrices to draw dilations.

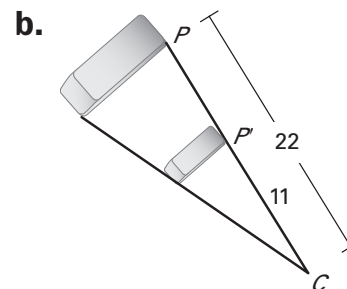
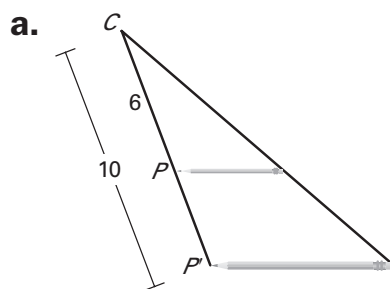
### Your Notes

#### VOCABULARY

**Scalar multiplication** Scalar multiplication is the process of multiplying each element of a matrix by a real number or *scalar*.

#### Example 1 Identify dilations

Find the scale factor of the dilation. Then tell whether the dilation is a *reduction* or an *enlargement*.



#### Solution

a. Because  $\frac{CP'}{CP} = \frac{10}{6}$ , the scale factor is  $k = \frac{5}{3}$ .

The image  $P'$  is an enlargement.

b. Because  $\frac{CP'}{CP} = \frac{11}{22}$ , the scale factor is  $k = \frac{1}{2}$ .

The image  $P'$  is a reduction.

- ✓ **Checkpoint** Complete the following exercise.

1. In a dilation,  $CP' = 4$  and  $CP = 20$ . Tell whether the dilation is a *reduction* or an *enlargement* and find its scale factor.

reduction;  $\frac{1}{5}$

**Example 2** Draw a dilation

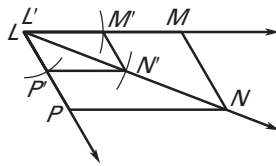
Draw and label  $\square LMNP$ . Then construct a dilation of  $\square LMNP$  with point  $L$  as the center of dilation and a scale factor of  $\frac{1}{2}$ .

**Solution**

**Step 1** Draw  $LMNP$ . Draw rays from  $L$  through vertices  $M$ ,  $N$ , and  $P$ .

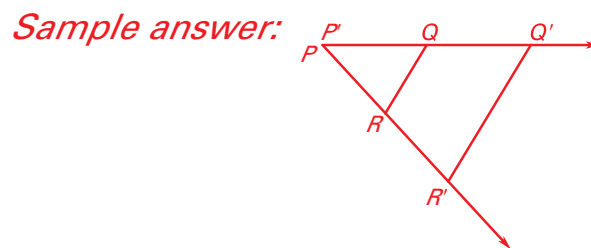
**Step 2** Open the compass to the length of  $\overline{LM}$ . Locate  $M'$  on  $\overrightarrow{LM}$  so  $LM' = \frac{1}{2}(LM)$ . Locate  $N'$  and  $P'$  the same way.

**Step 3** Add a second label  $L'$  to point  $L$ . Draw the sides of  $L'M'N'P'$ .



**Checkpoint** Complete the following exercise.

2. Draw and label  $\triangle PQR$ . Then construct a dilation of  $\triangle PQR$  with  $P$  as the center of dilation and a scale factor of 2.



## Your Notes

### Example 3 Scalar multiplication

Simplify the product:  $3 \begin{bmatrix} 0 & 5 & 4 \\ 2 & -2 & -1 \end{bmatrix}$ .

#### Solution

$$\begin{aligned} 3 \begin{bmatrix} 0 & 5 & 4 \\ 2 & -2 & -1 \end{bmatrix} &= \begin{bmatrix} \underline{3(0)} & \underline{3(5)} & \underline{3(4)} \\ \underline{3(2)} & \underline{3(-2)} & \underline{3(-1)} \end{bmatrix} && \text{Multiply each} \\ & & & \text{element in the} \\ & & & \text{matrix by } \underline{3}. \\ &= \begin{bmatrix} \underline{0} & \underline{15} & \underline{12} \\ \underline{6} & \underline{-6} & \underline{-3} \end{bmatrix} && \text{Simplify.} \end{aligned}$$

✔ **Checkpoint** Simplify the product.

$$\begin{aligned} 3. \quad 4 \begin{bmatrix} -6 & 3 & 2 \\ 5 & -1 & 4 \end{bmatrix} \\ \begin{bmatrix} -24 & 12 & 8 \\ 20 & -4 & 16 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} 4. \quad -3 \begin{bmatrix} 5 & -1 & -2 \\ -2 & 0 & 4 \end{bmatrix} \\ \begin{bmatrix} -15 & 3 & 6 \\ 6 & 0 & -12 \end{bmatrix} \end{aligned}$$

### Example 4 Use scalar multiplication in a dilation

The vertices of quadrilateral  $ABCD$  are  $A(-3, 0)$ ,  $B(0, 6)$ ,  $C(3, 6)$ , and  $D(3, 3)$ . Use scalar multiplication to find the image of  $ABCD$  after a dilation with its center at the origin and a scale factor of  $\frac{1}{3}$ . Graph  $ABCD$  and its image.

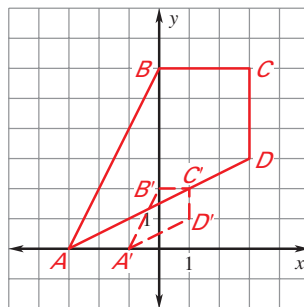
#### Solution

$$\frac{1}{3} \begin{bmatrix} A & B & C & D \\ -3 & 0 & 3 & 3 \\ 0 & 6 & 6 & 3 \end{bmatrix} = \begin{bmatrix} A' & B' & C' & D' \\ -1 & 0 & 1 & 1 \\ 0 & 2 & 2 & 1 \end{bmatrix}$$

Scale factor

Polygon matrix

Image matrix



## Your Notes

### Example 5 Find the image of a composition

The vertices of  $\triangle KLM$  are  $K(-3, 0)$ ,  $L(-2, 1)$ , and  $M(-1, -1)$ . Find the image of  $\triangle KLM$  after the given composition.

Translation:  $(x, y) \rightarrow (x + 4, y + 2)$

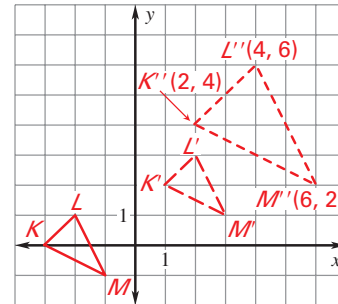
Dilation: centered at the origin with a scale factor of 2

#### Solution

**Step 1** Graph the preimage  $\triangle KLM$  in the coordinate plane.

**Step 2** Translate  $\triangle KLM$  4 units to the right and 2 units up. Label it  $\triangle K'L'M'$ .

**Step 3** Dilate  $\triangle K'L'M'$  using the origin as the center and a scale factor of 2 to find  $\triangle K''L''M''$ .



✓ **Checkpoint** Complete the following exercises.

5. The vertices of  $\triangle RST$  are  $R(-4, 3)$ ,  $S(-1, -2)$ , and  $T(2, 1)$ . Use scalar multiplication to find the vertices of  $\triangle R'S'T'$  after a dilation with its center at the origin and a scale factor of 2.

$$R(-8, 6), S(-2, -4), T(4, 2)$$

6. A segment has the endpoints  $C(-2, 2)$  and  $D(2, 2)$ . Find the image of  $\overline{CD}$  after a  $90^\circ$  rotation about the origin followed by a dilation with its center at the origin and a scale factor of 2.

$$C'(-4, -4), D'(-4, 4)$$

### Homework