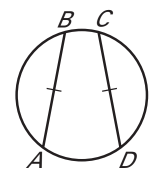
10.3 Apply Properties of Chords

Goal • Use relationships of arcs and chords in a circle.

Your Notes

THEOREM 10.3

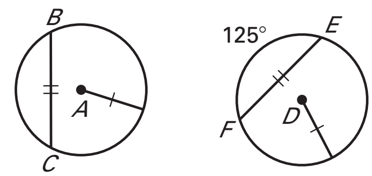
In the same circle, or in congruent circles, two minor arcs are congruent if and only if their corresponding chords are congruent.



$\overset{\frown}{AB} \cong \overset{\frown}{CD}$ if and only if $\overline{AB}$ ≅ $\overline{CD}$.

Example 1 ***Use congruent chords to find an arc measure***

In the diagram, $\odot A \cong \odot D$, $\overline{BC} \cong \overline{EF}$, and $m\overset{\frown}{EF} = 125°$. Find $m\overset{\frown}{BC}$.



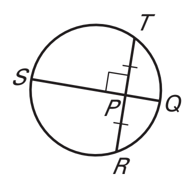
Solution

Because $\overline{BC}$ and $\overline{EF}$ are congruent chords in congruent circles, the corresponding minor arcs $\overset{\frown}{BC}$ and $\overset{\frown}{EF}$ are congruent.

So, $m\overset{\frown}{BC} = m\overset{\frown}{EF} =$ 125°.

THEOREM 10.4

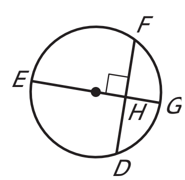
If one chord is a perpendicular bisector of another chord, then the first chord is a diameter.



If $\overline{QS}$ is a perpendicular bisector of $\overline{TR}$, then $\overline{QS}$ is a diameter of the circle.

THEOREM 10.5

If a diameter of a circle is perpendicular to a chord, then the diameter bisects the chord and its arc.

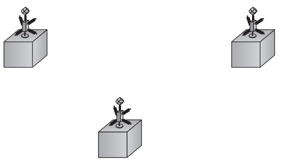


If $\overline{EG}$ is a diameter and $\overline{EG} \perp \overline{DF}$, then $\overline{HD} \cong \overline{HF}$ and $\overset{\frown}{GD}$ ≅ $\overset{\frown}{GF}$.

Your Notes

Example 2 *Use perpendicular bisectors*

Journalism A journalist is writing a a story about three sculptures, arranged as shown at the right. Where should the journalist place a camera so that it is the same distance from each sculpture?

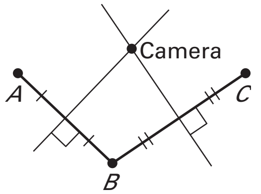


Solution

Step 1 **Label** the sculptures *A*, *B*, and *C*. Draw segments $\overline{AB}$ and $\overline{BC}$.

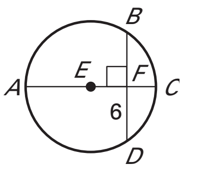
Step 2 **Draw** the _perpendicular bisectors_ of $\overline{AB}$ and $\overline{BC}$. By _Theorem 10.4_, these are diameters of the circle containing *A*, *B*, and *C*.

Step 3 **Find** the point where these bisectors _intersect_. This is the center of the circle through *A*, *B*, and *C*, and so it is _equidistant_ from each point.



Example 3 *Use a diameter*

Use the diagram of ⊙*E* to find the length of $\overline{BD}$. Tell what theorem you use.

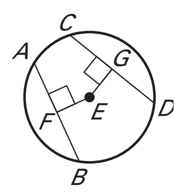


Solution

Diameter $\overline{AC}$ is _perpendicular_ to $\overline{BD}$. So, by Theorem 10.5, $\overline{AC}$ _bisects_ $\overline{BD}$, and BF = _*DF*_.
Therefore, $BD = 2($_*DF*_$) = 2($_6_$) =$ _12_.

THEOREM 10.6

In the same circle, or in congruent circles, two chords are congruent if and only if they are equidistant from the center.

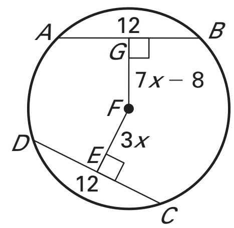


$\overline{AB} \cong \overline{CD}$ if and only if _*EF*_ = _*EG*_.

Your Notes

Example 4 *Use Theorem 10.6*

In the diagram of ⊙F, $AB = CD = 12$. Find EF.



Solution

Chords $\overline{AB}$ and $\overline{CD}$ are congruent, so by Theorem 10.6 they are equidistant from F. Therefore, EF = GF.

EF = GF — **Use Theorem 10.6.**

$3x$ = $7x - 8$ — **Substitute.**

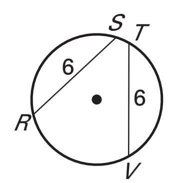
x = 2 — **Solve for x.**

So, $EF = 3x = 3($ 2 $)$ = 6.

✔ ***Checkpoint*** **Complete the following exercises.**

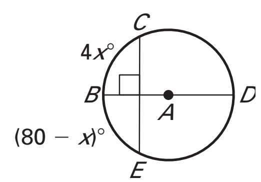
1. If $m\overset{\frown}{TV} = 121°$, find $m\overset{\frown}{RS}$.

$m\overset{\frown}{RS} = 121°$



2. Find the measures of $\overset{\frown}{CB}$, $\overset{\frown}{BE}$, and $\overset{\frown}{CE}$.

$m\overset{\frown}{CB} = 64°$, $m\overset{\frown}{BE} = 64°$, $m\overset{\frown}{CE} = 128°$



3. In the diagram in Example 4, suppose $AB = 27$ and $EF = GF = 7$. Find CD.

$CD = 27$

Homework