10.4 Use Inscribed Angles and Polygons

Goal • Use inscribed angles of circles.

Your Notes

VOCABULARY

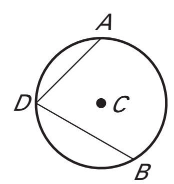
Inscribed angle An inscribed angle is an angle whose vertex is on a circle and whose sides contain chords of the circle.

Intercepted arc The arc that lies in the interior of an inscribed angle and has endpoints on the angle is called the intercepted arc of the angle.

Inscribed polygon A polygon is an inscribed polygon if all of its vertices lie on a circle.

Circumscribed circle A circumscribed circle is a circle that contains the vertices of an inscribed polygon.

THEOREM 10.7: MEASURE OF AN INSCRIBED ANGLE THEOREM

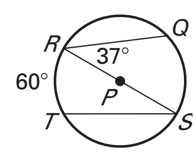


The measure of an inscribed angle is one half the measure of its intercepted arc.

$m\angle ADB = \frac{1}{2}\ m\overset{\frown}{AB}$

Example 1 *Use inscribed angles*

Find the indicated measure in $\odot P$.



a. $m\angle S$ **b.** $m\overset{\frown}{RQ}$

Solution

a. $m\angle S = \frac{1}{2}\ m\overset{\frown}{RT} = \frac{1}{2}(60°) = 30°$

b. $m\overset{\frown}{QS} = 2m\angle R = 2 \cdot 37° = 74°$.

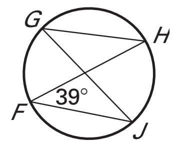
Because $\overset{\frown}{RQS}$ is a semicircle,

$m\overset{\frown}{RQ} = 180° - m\overset{\frown}{QS} = 180° - 74° = 106°$.

Your Notes

Example 2 *Find the measure of an intercepted arc*

Find $m\widehat{HJ}$ and $m\angle HGJ$. What do you notice about $\angle HGJ$ and $\angle HFJ$?



Solution

From Theorem 10.7, you know that

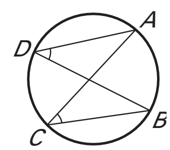
$m\widehat{HJ} = 2m\angle$ HFJ $= 2($ 39° $) =$ 78° .

Also, $m\angle HGJ = \frac{1}{2}$ $m\widehat{HJ}$ $= \frac{1}{2}($ 78° $) =$ 39° .

So $\angle HGJ$ ≅ $\angle HFJ$.

THEOREM 10.8

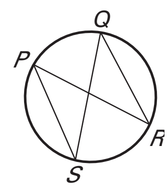
If two inscribed angles of a circle intercept the same arc, then the angles are congruent.



$\angle ADB \cong \angle$ ACB

Example 3 *Use Theorem 10.8*

Name two pairs of congruent angles in the figure.

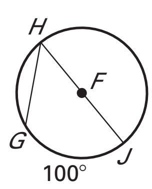


Solution

Notice that $\angle QRP$ and $\angle$ QSP intercept the same arc, and so $\angle QRP \cong \angle$ QSP by Theorem 10.8. Also, $\angle RQS$ and $\angle$ RPS intercept the same arc, so $\angle RQS \cong \angle$ RPS .

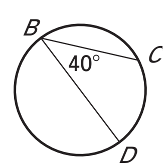
Checkpoint **Find the indicated measure.**

1. $m\angle GHJ$



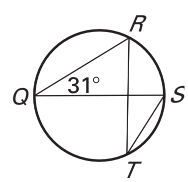
$m\angle GHJ = 50°$

2. $m\widehat{CD}$



$m\widehat{CD} = 80°$

3. $m\angle RTS$

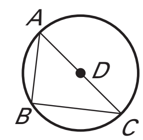


$m\angle RTS = 31°$

Your Notes

THEOREM 10.9

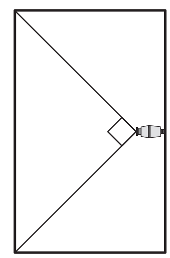
If a right triangle is inscribed in a circle, then the hypotenuse is a diameter of the circle. Conversely, if one side of an inscribed triangle is a diameter of the circle, then the triangle is a right triangle and the angle opposite the diameter is the right angle.



$m\angle ABC = 90^\circ$ if and only if $\overline{AC}$ is a diameter of the circle.

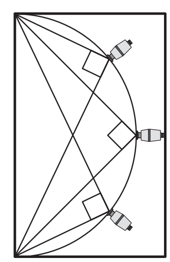
Example 4 ***Use a circumscribed circle***

Security A security camera rotates 90° and needs to be able to view the width of a wall. The camera is placed in a spot where the only thing viewed when rotating is the wall. You want to change the camera's position. Where else can it be placed so that the wall is viewed in the same way?



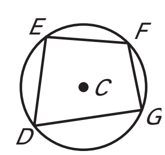
Solution

From Theorem 10.9, you know that if a right triangle is inscribed in a circle, then the hypotenuse of the triangle is a diameter of the circle. So, draw the circle that has the width of the wall as a diameter. The wall fits perfectly with your camera's 90° rotation from any point on the semicircle in front of the wall.



THEOREM 10.10

A quadrilateral can be inscribed in a circle if and only if its opposite angles are supplementary.



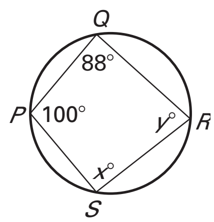
D, E, F, and G lie on $\odot C$ if and only if $m\angle D + m\angle F = m\angle E + m\angle G = 180^\circ$.

Your Notes

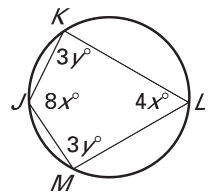
Example 5 *Use Theorem 10.10*

Find the value of each variable.

a.



b.



Solution

a. *PQRS* is inscribed in a circle, so opposite angles are supplementary.

$m\angle P + m\angle R =$ 180° $m\angle Q + m\angle S =$ 180°

$100° + y° =$ 180° $88° + x° =$ 180°

$y =$ 80 $x =$ 92

b. *JKLM* is inscribed in a circle, so opposite angles are supplementary.

$m\angle J + m\angle L =$ 180° $m\angle K + m\angle M =$ 180°

$8x° + 4x° =$ 180° $3y° + 3y° =$ 180°

$12x =$ 180 $6y =$ 180

$x =$ 15 $y =$ 30

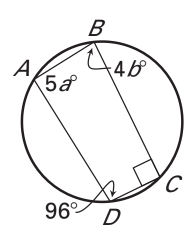
✔ ***Checkpoint*** **Complete the following exercises.**

4. A right triangle is inscribed in a circle. The radius of the circle is 5.6 centimeters. What is the length of the hypotenuse of the right triangle?

11.2 centimeters

5. Find the values of *a* and *b*.

$a = 18$, $b = 21$



Homework