10.6 Find Segment Lengths in Circles

Goal • Find segment lengths in circles.

Your Notes

VOCABULARY

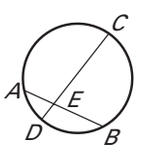
Segments of a chord When two chords intersect in the interior of a circle, each chord is divided into two segments called segments of the chord.

Secant segment A secant segment is a segment that contains a chord of a circle, and has exactly one endpoint outside the circle.

External segment An external segment is the part of a secant segment that is outside the circle.

THEOREM 10.14: SEGMENTS OF CHORDS THEOREM

If two chords intersect in the interior of a circle, then the product of the lengths of the segments of one chord is equal to the product of the lengths of the segments of the other chord.



$EA \cdot$ *EB* $= EC \cdot$ *ED*

Example 1 ***Find lengths using Theorem 10.14***

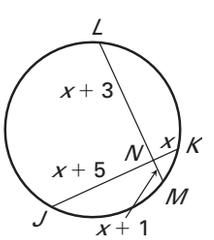
Find *ML* and *JK*.

$NK \cdot NJ =$ *NL* $\cdot$ *NM*

$x \cdot (x + 5) = (x + 3) \cdot (x + 1)$

$x^2 + 5x = x^2 + 4x + 3$

$x = 3$



Find *ML* and *JK* by substitution.

$ML = (x + 1) + (x + 3)$

$= 3 + 1 + 3 + 3$

$= 10$

$JK = x + (x + 5)$

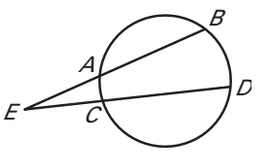
$= 3 + 3 + 5$

$= 11$

Your Notes

THEOREM 10.15: SEGMENTS OF SECANTS THEOREM

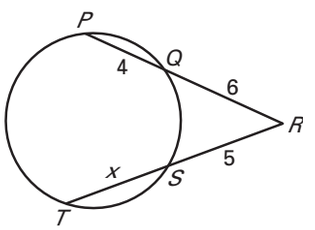
If two secant segments share the same endpoint outside a circle, then the product of the lengths of one secant segment and its external segment equals the product of the lengths of the other secant segment and its external segment.



$EA \cdot \underline{EB} = EC \cdot \underline{ED}$

Example 2 ***Use Theorem 10.15***

Find the value of *x*.



Solution

$RQ \cdot RP = RS \cdot RT$ **Use Theorem** 10.15 **.**

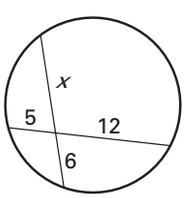
$\underline{6} \cdot (\underline{6} + \underline{4}) = \underline{5} \cdot (x + \underline{5})$ **Substitute.**

$\underline{60} = \underline{5}x + \underline{25}$ **Simplify.**

$\underline{7} = x$ **Solve for *x*.**

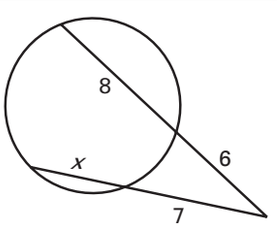
Checkpoint **Find the value of *x*.**

1.



$x = 10$

2.

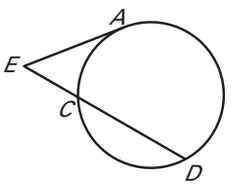


$x = 5$

Your Notes

THEOREM 10.16: SEGMENTS OF SECANTS AND TANGENTS THEOREM

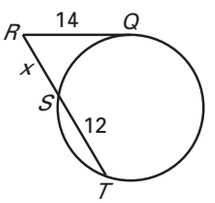
If a secant segment and a tangent segment share an endpoint outside a circle, then the product of the lengths of the secant segment and its external segment equals the square of the length of the tangent segment.



$EA^2 =$ *EC* • *ED*

Example 3 ***Find lengths using Theorem 10.16***

Use the figure at the right to find *RS*.



Solution

$RQ^2 = RS \cdot RT$ — **Use Theorem** 10.16 **.**

$14^2 = x \cdot (x + 12)$ — **Substitute.**

$196 = x^2 + 12x$ — **Simplify.**

$0 = x^2 + 12x - 196$ — **Write in standard form.**

$x = \dfrac{-12 \pm \sqrt{12^2 - 4(1)(-196)}}{2(1)}$ — **Use quadratic formula.**

$x = -6 \pm 2\sqrt{58}$ — **Simplify.**

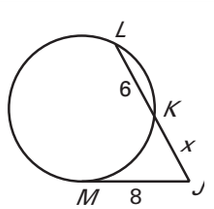
Lengths cannot be negative, so use the positive solution.

So, $x =$ $-6 + 2\sqrt{58}$ $\approx$ 9.23, and $RS \approx$ 9.23.

Checkpoint **Complete the following exercise.**

3. Use the figure at the right to find *JK*.

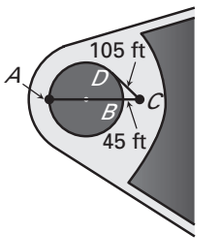
$JK = -3 + \sqrt{73}$



Your Notes

Example 4 ***Solve a real-world problem***

Fountain You are standing at point *C*, 45 feet from the Point State Park fountain in Pittsburgh, PA. The distance from you to a point of tangency on the fountain is 105 feet. Find the distance *CA* between you and your friend at point *A*.



Solution

$\underline{CB} \cdot CA = \underline{CD^2}$ **Use Theorem 10.16.**

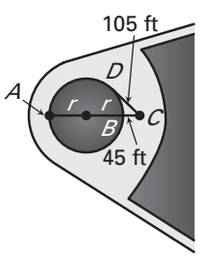
$\underline{45} \cdot CA = \underline{105^2}$ **Substitute.**

$CA = \underline{245}$ **Solve for *CA*.**

You are 245 feet from your friend.

✔ ***Checkpoint*** **Complete the following exercise.**

4. In Example 4, suppose $\overline{AB}$ is a diameter of the fountain. Use the diagram below and Theorem 10.16 to find the radius of the fountain.



100 ft

Homework