10.7 Write and Graph Equations of Circles

Goal • Write equations of circles in the coordinate plane.

Your Notes

VOCABULARY

Standard equation of a circle The standard equation of a circle with center (h, k) and radius r is $(x - h)^2 + (y - k)^2 = r^2$.

Example 1 *Write an equation of a circle*
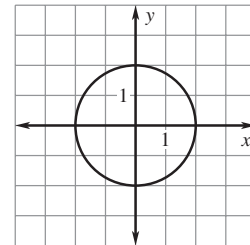
Write the equation of the circle shown.

Solution

The radius is __2__ and the center is at __the origin__.

$x^2 + y^2 = $ __r__ ^{2} **Equation of circle**

$x^2 + y^2 = $ __2__ ^{2} **Substitute.**
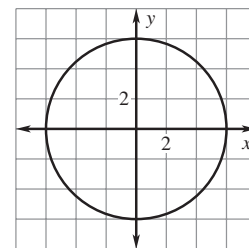
$x^2 + y^2 = $ __4__   **Simplify.**

The equation of the circle is $x^2 + y^2 = $ __4__ .

✔ *Checkpoint* **Complete the following exercise.**

1. Write an equation of the circle shown.

$x^2 + y^2 = 36$

STANDARD EQUATION OF A CIRCLE

The standard equation of a circle with center (h, k) and radius r is:

$(x - h)^2 + (y - k)^2 = r^2$

Example 2 *Write the standard equation of a circle*

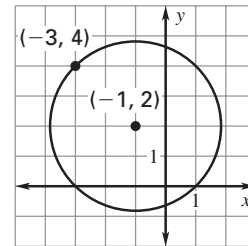Write the standard equation of a circle with center $(0, -5)$ and radius 3.7.

$(x - h)^2 + (y - k)^2 = r^2$ **Standard equation of a circle**

$(x - \underline{\ 0\ })^2 + (y - (\underline{\ -5\ }))^2 = \underline{\ 3.7\ }^2$ **Substitute.**

$x^2 + (y + \underline{\ 5\ })^2 = \underline{\ 13.69\ }$ **Simplify.**

Example 3 *Write the standard equation of a circle*

The point $(-3, 4)$ is on a circle with center $(-1, 2)$. Write the standard equation of the circle.

Solution

To write the standard equation, you need to know the values of h, k, and r. To find r, find the distance between the __center__ and the point $(-3, 4)$ on the circle.

$r = \sqrt{[-3 - (\underline{\ -1\ })]^2 + (\underline{\ 4\ } - 2)^2}$ **Distance formula**

$= \sqrt{(\underline{\ -2\ })^2 + \underline{\ 2\ }^2}$ **Simplify.**

$= \underline{\ 2\sqrt{2}\ }$ **Simplify.**

Substitute $(h, k) = (-1, 2)$ and $r = \underline{\ 2\sqrt{2}\ }$ into the standard equation of a circle.

$(x - h)^2 + (y - k)^2 = r^2$ **Standard equation of a circle**

$(x - (\underline{\ -1\ }))^2 + (y - \underline{\ 2\ })^2 = (\underline{\ 2\sqrt{2}\ })^2$ **Substitute.**

$(x + \underline{\ 1\ })^2 + (y - \underline{\ 2\ })^2 = \underline{\ 8\ }$ **Simplify.**

The standard equation of the circle is $(x + \underline{\ 1\ })^2 + (y - \underline{\ 2\ })^2 = \underline{\ 8\ }$.

Example 4   *Graph a circle*

The equation of a circle is $(x - 2)^2 + (y + 3)^2 = 16$. Graph the circle.

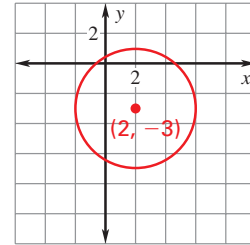> If you know the equation of a circle, you can graph the circle by identifying its center and radius.

Solution

Rewrite the equation to find the center and radius.

$$(x - 2)^2 + (y + 3)^2 = 16$$

$$(x - 2)^2 + [y - (\underline{-3})]^2 = \underline{4^2}$$

The center is ($\underline{2}$, $\underline{-3}$) and the radius is $\underline{4}$. Use a compass to graph the circle.

(2, −3)

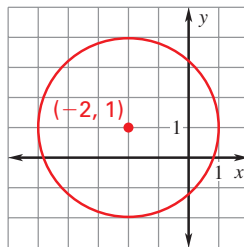✔ *Checkpoint*   **Complete the following exercises.**

2. Write the standard equation of a circle with center $(-3, -5)$ and radius 6.1.

$(x + 3)^2 + (y + 5)^2 = 37.21$

3. The point $(-1, 2)$ is on a circle with center $(3, -3)$. Write the standard equation of the circle.

$(x - 3)^2 + (y + 3)^2 = 41$

4. The equation of a circle is $(x + 2)^2 + (y - 1)^2 = 9$. Graph the circle.

(−2, 1)

Example 5 *Use graphs of circles*

Time Capsule You bury a time capsule and use a grid to write directions for finding it. Use the following measurements to find the burial location of the time capsule.

- The capsule is about **11** feet from the oak tree at *A*(0, 0).

- The capsule is 8 feet from the flagpole at *B*(0, 8).

- The capsule is 4 feet from the mailbox at *C*(−12, 8).

Solution

The set of all points equidistant from a given point is a circle, so the burial location is located on each of the following circles.
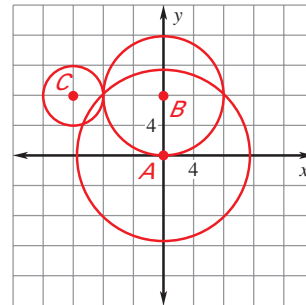
⊙*A* with center (0 , 0) and radius 11

⊙*B* with center (0 , 8) and radius 8

⊙*C* with center (−12 , 8) and radius 4

To find the burial location, graph the circles on a graph where units are measured in feet. Estimate the point of intersection of all three circles.

The burial location is at about (−8 , 8).

✔ *Checkpoint* **Complete the following exercise.**

5. In Example 4, suppose the mailbox is at *C*(12, 8) and the time capsule is 4 feet away. Find the burial location of the time capsule.

(8, 8)

Homework