Experiment 1

Vector Addition of Forces

Objective:

- To test the hypothesis that forces combine by the rules of vector addition and that the net force acting on an object at rest is zero.

Equipment:

- Five spring balances
- Five 1-2 kg weights used for anchors
- Small washer
- Large sheet of paper
- Ruler, protractor, right triangle
- Scientific calculator (with sin, cos & tan functions) or Mathcad

Physical principles:

Definitions of Sine, Cosine, and Tangent of an Angle

Consider one of the acute (less than 90°) angles, *a*, of the right triangle shown in figure 1. As a result of where they reside, the three sides of the triangle are called the opposite side, adjacent side and hypotenuse. The two sides that make up the right angle (exactly 90°) are always the adjacent side and the opposite side. As a result, the length of the hypotenuse is always greater than the length of each of the other two sides but less than the sum of the lengths of the other two sides. The size of the angle *a* can be related to the length of the three sides of the right triangle by the use of the trigonometric functions Sine, Cosine and Tangent, abbreviated *sin*, *cos* and *tan*, respectively. They are defined as shown below.

$$sin(a) \quad \frac{opposite}{hypotenuse}$$

$$cos(a) \quad \frac{adjacent}{hypotenuse} \qquad \textbf{(1)}$$

$$tan(a) \quad \frac{opposite}{adjacent}$$

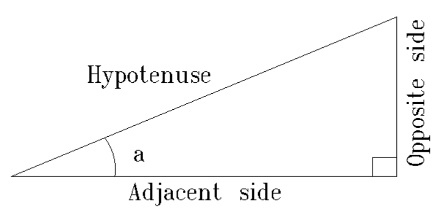

Figure 1 *The sides of a right triangle.*

Vector Addition

Polygon method - Vectors may be added graphically by repositioning each one so that its tail coincides with the head of the previous one (see figure. 2). The resultant (sum of the forces) is the vector drawn from the tail of the first vector to the head of the last. The magnitude (length) and angle of the resultant is measured with a ruler and a protractor, respectively. Note: In order to measure the angle, a set of axes must first be defined.

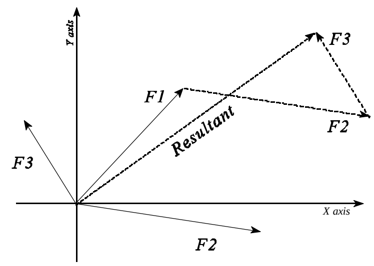

Figure 2 *Vector addition by the polygon method.*

Component method - Vectors may be added by selecting two perpendicular directions called the X and Y axes, and projecting each vector on to these axes. This process is called the resolution of a vector into components in these directions. If the angle *a* that the vector makes from the positive X axis, is used (see figure 3), these components are given by

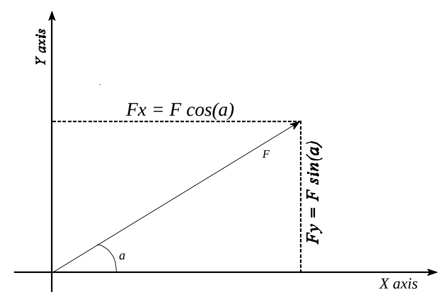

Figure 3 *Finding the two perpendicular components of a vector.*

$$F_x = F \cdot \cos(a)$$
$$F_y = F \cdot \sin(a) \qquad \textbf{(2)}$$

The X component of the resultant is the sum of the X components of the vectors being added, and similarly for the Y component.

$$R_x = \sum F_x$$
$$R_y = \sum F_y \qquad \textbf{(3)}$$

The angle that the resultant makes with the X axis is given by

$$a = \arctan\left(\frac{R_y}{R_x}\right) \qquad \textbf{(4)}$$

and the magnitude is given by

$$R = \sqrt{R_x^2 + R_y^2} \qquad \textbf{(5)}$$

Equilibrium Conditions

Newton's second law predicts that a body will not accelerate when the net force acting on it is zero. So, for an object to be at rest, the resultant force acting on it <u>must</u> be zero. In equation form, the above statement can be written

$$\sum \vec{F} \quad 0 \qquad \textbf{(6)}$$

Thus, if four forces act on an object at rest, the following relationship has to be satisfied.

$$\vec{F}_1 \;\; \vec{F}_2 \;\; \vec{F}_3 \;\; \vec{F}_4 \;\; 0 \qquad \textbf{(7)}$$

An equivalent statement is

$$\vec{F}_4 \quad (\vec{F}_1 \;\; \vec{F}_2 \;\; \vec{F}_3) \qquad \textbf{(8)}$$

so that $\vec{F}_4$ is equal in magnitude and opposite in direction to the resultant of the other three forces.

Procedure:

Set up the following situations so that in each case the magnitudes of the forces are unequal.

1. Attach three strings about 12 cm long to the small washer and connect the other end to spring balances, to the end connected directly to the center force indicating shaft. Connect string loops, about 8 cm long, to the other end of the spring balances and wrap these loops around the 1-2 kg weights (see figure 4). You will need to make sure that when there is no load on the spring balance the scale reads zero. If it does not, you will need to adjust it by sliding the metal tab at the top of the device.

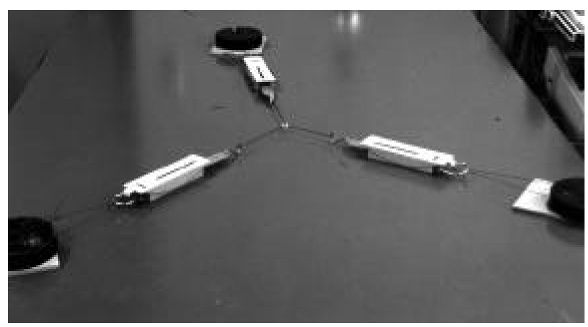

Figure 4 *Sample setup of three forces acting on a small washer at equilibrium.*

a) Move the weights so that the angle between forces F_1 and F_2 is 90° (see figure 5). On a paper (as large as .3 by .3 m, if possible) draw lines parallel to its edges and intersecting near its center. These lines will act as the X and Y axes, described in the Physical Principles section. Position the paper so that the origin of the axes is right under the small washer, with the forces F_1 and F_2 along the two lines. Tape the sheet of paper to the table. Use a pencil to mark

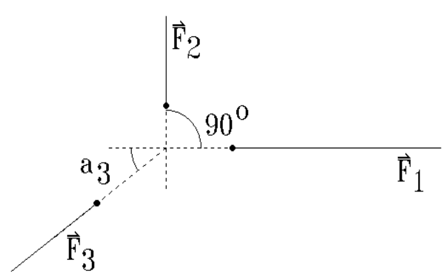

Figure 5 *Procedure 1.a) setup.*

two points at opposite ends of the string supplying the force F_1. By connecting these two points, draw a line below the string showing the direction of the force. Following the same procedure, draw the direction of the other two forces. Record the weight on each string in Newtons. For those spring balances calibrated in grams, convert the scale readings by multiplying by $9.80*10^{-3}$ N/g. Place arrows on your lines in the direction of the force exerted by the spring balances. Select your X axis to be along the line of force F_1. Add the vectors for F_1 and F_2 both graphically (polygon method) and with trigonometry (component method). Compare the magnitude of the resultant with that of the force, F_3 for both solutions. Using a protractor, measure a_3 and compare it with the similarly measured angle of your graphical addition and your trigonometrically computed angle. Do your measurements satisfy the requirements of Newton's second law?

b) Repeat as outlined in part (a) using the component method only, but with the angle between F_1 and F_2 at about 120°. Do your measurements satisfy the requirements of Newton's second law?

2. Repeat step 1a, using only the component addition method with 4 spring balances (see figure 6). Draw the forces F_1, F_2, F_3, and F_4 approximately as illustrated. Find and add the components of F_1, F_2, and F_3. Compute the magnitude and direction of the sum of these forces and compare your result with a_4 and F_4. Do your measurements satisfy the requirements of Newton's second law?

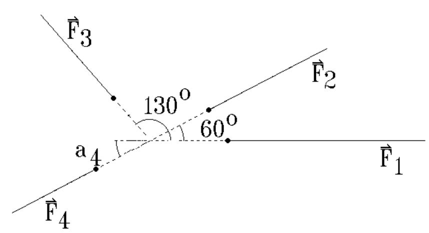

Figure 6 *Procedure 2 setup.*

3. If time permits, for extra credit, repeat as in step 2 using 5 forces extended approximately as illustrated in figure 7. Do your measurements satisfy the requirements of Newton's second law?

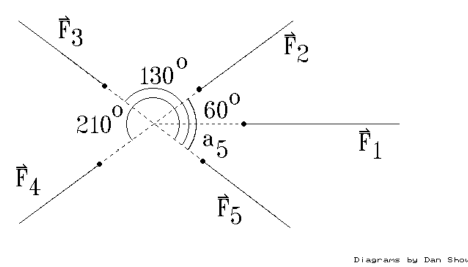

Figure 7 *Procedure 3 setup (extra credit).*

Recording data:

Part 1a.

Table 1 Polygon Method

Force	Magnitude (N)	Angle (°)
Force 1		
Force 2		
Force 3		
Resultant of 1 & 2		

Table 2 Component Method

Direction	Force 1	Force 2	Resultant
X			
Y			

Magnitude of resultant = ________________________

Angle of resultant = ________________________

Part 1b.

Table 3 Component Method

Direction	Force 1	Force 2	Resultant
X			
Y			

Magnitude of resultant = ________________________

Angle of resultant = ____________________________

Part 2

Table 4 Component Method

Direction	Force 1	Force 2	Force 3	Resultant
X				
Y				

Magnitude of resultant = ________________________

Angle of resultant = ____________________________

Part 3 (Optional - Extra credit)

Table 4 Component Method

Direction	Force 1	Force 2	Force 3	Force 4	Resultant
X					
Y					

Magnitude of resultant = ________________________

Angle of resultant = ____________________________