Math 192 Review for Final 3 Name:

Final exam is 20% of the grade so that it will affect your grade significantly. Please prepare well for the final and don't forget to go over the past exams.

1. A force of 8 dynes is required to stretch a spring from its natural length of 10 cm to a length of 15 cm. How much work is done

(a) in stretching the spring to a length of 25 cm?

(b) in stretching the spring from a length of 20 cm to a length of 25cm?

2. Solve the following differential equation.

(a) $\frac{dy}{dx} - 4y = 0$ (b) $\frac{dy}{dx} - 4xy = 0$ (c) $\frac{dy}{dx} = y^2 - 4y + 3$

3. In paleontology, the phenomenon of a radioactive decay is commonly used to date fossil remains. This method uses the fact the rate of decay $\frac{dA}{dt}$ of an element will be proportional to the amount A of the radioactive element that exists at time t.

(a) Determine a differential equation that A(t) must satisfy.

(b) What can you say about $A(t)$ as $t \to \infty$?

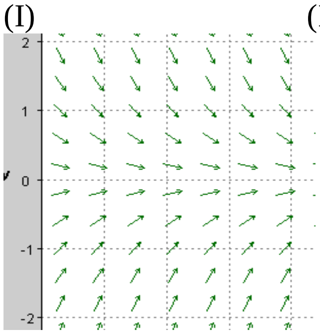
(c) Find a solution to your differential equation.

(d) How long will it take for half of the original amount of the radioactive element to decay?

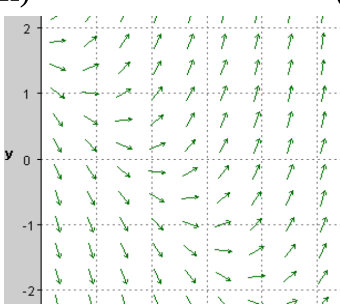
4. Match the direction field with the differential equation given below.

(a) $\frac{dy}{dx} = 1$ (b) $\frac{dy}{dx} = x^2$ (c) $\frac{dy}{dx} = x + y$ (d) $\frac{dy}{dx} = -y$

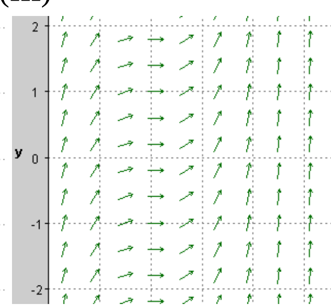
(I)



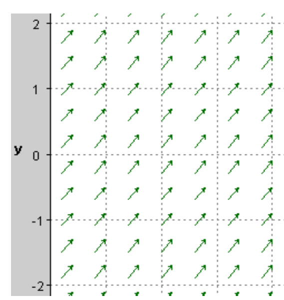
(II)



(III)



(IV)



5. Newton's Law of Cooling states that the rate at which a body changes temperature is proportional to the difference between its temperature and the temperature of the surrounding medium. Suppose that a body has an initial temperature of 250 $^\circ F$ and that after one hour the temperature is 200$^\circ F$. Assuming that the surrounding air is kept at a constant temperature of 72$^\circ F$, determine the temperature of the body at time t.

6. Radium has a half life of 1600 years. How many years does it take for 90% of a given amount of radium to decay?

7. Determine the limit of the sequence below.

(a) $a_n = \frac{(-1)^n}{\sqrt{n}}$ (b) $a_n = \frac{(-2)^n}{n}$ (c) $\frac{3\cos n + 1}{n}$ (d) 1.0001^n (e) $\frac{n\cos n}{n^2+1}$

8. Find the sum of the following series if exists.

(a) $\sum_{n=1}^{\infty} \frac{1}{n(n+2)}$ (b) $\sum_{n=0}^{\infty} 3[(\frac{1}{2})^n + (-\frac{1}{2})^n]$ (c) $\sum_{n=4}^{\infty} \ln(\frac{n+1}{n})$ (d) $\sum_{n=1}^{\infty} \frac{n-1}{2n+1}$ (e) $\sum_{n=1}^{\infty} \frac{1}{e^{2n}}$

9. Which of the following series converge? If possible, find the sum. Explain your answer.

(a) $\sum_{n=1}^{\infty}\frac{\pi^n}{4^n}$ (b) $\sum_{n=1}^{\infty}\frac{1}{n\sqrt{n}}$ (c) $\sum_{n=2}^{\infty}\frac{1}{n\ln n}$ (d) $\sum_{n=1}^{\infty}\frac{1}{(n+1)^3}$ (e) $\sum_{n=1}^{\infty}\frac{1+\sin^2 n}{5^n}$ (f) $\sum_{n=1}^{\infty}\frac{1}{n+1}$ (g) $\sum_{n=1}^{\infty}\frac{(-1)^n}{\sqrt{n+1}}$

(h) $\sum_{n=1}^{\infty}(\frac{1}{n}-\frac{1}{n+1})$ (i) $\sum_{n=1}^{\infty}\frac{(-1)^n n^2}{n^2+1}$ (j) $\sum_{n=1}^{\infty}\frac{1}{2+3^n}$ (k) $\sum_{n=1}^{\infty}\frac{\ln n}{(n+1)^3}$

10. Approximate the sum $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n^4}$ with error < 0.001.

11. Consider the series $\sum_{n=1}^{\infty}\frac{(-1)^{n-1}n}{4^n}$.

(a) Show that the series is absolutely convergent. Is it convergent, then?
(b) Calculate the sum of the first three terms to approximate the sum of the series.
(c) Estimate the error involved in the approximation from part (b).

12. Find the radius of convergence of the following.

(a) $\sum_{n=0}^{\infty}\frac{n!}{4^n}(x+3)^n$ (b) $\sum_{n=0}^{\infty}\frac{x^n}{3n+1}$ (c) $\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n+1}}{(2n+1)!}$ (what is this series?)

13. Find a power series representation for the following function.

(a) $f(x)=\frac{x}{x+5}$ (b) $\int\frac{x}{x^3+1}dx$

14. Find the sum of $\sum_{n=1}^{\infty}\frac{3^n}{5^n n!}$.

15. Find the Maclaurin series for the following.

(a) $\ln(1+x)$ (b) xe^x (c) $(1+x)^{1/3}$

16. Replace the following polar equations by equivalent Cartesian equations, and identify their graphs.

(a) $r^2=4r\cos\theta$ (b) $r=\frac{4}{2\cos\theta-\sin\theta}$ (c) $\theta=\frac{\pi}{6}$

17. Find the parametric equations and parameter intervals for the motion of a particle that starts at (a,0) and traces the ellipse $(x^2/a^2)+(y^2/b^2)=1$ (a) once clockwise (b) twice counterclockwise.

18. Find the equation of the line passing thru the points A(-1,1,0) and B(0,2,3).

19. Find the equation of the plane passing thru the points A(0,1,0) , B(1,2,-1) and C(0,-1,2).

20. Find the first order partial derivatives of the function $f(x,y)=x\ln(\frac{x+y}{x-y})$.

Answer key

1. (a) 180 dynes-cm (b) 100 dynes-cm

2. (a) $y = e^{4x} \cdot C$ (b) $y = C \cdot e^{2x^2}$ (c) $\ln\left|\frac{y-3}{y-1}\right| = 2x + C$

3. (a) $\frac{dA}{dt} = -kA, k > 0$ (b) it approaches to 0 since $\frac{dA}{dt} < 0$ (c) $A(t) = Ae^{-kt}$ (d) $t = \frac{\ln 2}{k}$

4. (I) d (II) c (III) b (IV) a

5. $\frac{dT}{dt} = k(T-72), T(0) = 250^\circ F$,

$T(1) = 200^\circ F \Rightarrow T(t) = 72 + 178(\frac{128}{178})^t = 72 + 178e^{-0.32975t} = 72 + 178 \cdot 0.7191^t$

6. $\frac{1600 \ln 10}{\ln 2}$

7. (a) 0 (b) divergent (c) 0 (d) divergent (e) 0

8. (a) ¾ (b) 8 (c), (d) divergent (e) $\frac{1}{e^2 - 1}$

9. (a) converges and sum is $\frac{\pi}{4-\pi}$ (b) converges since it is a p-series, p=3/2>1 (c) diverges by integral test (d) converges since it is a p-series, p =3 >1 (e) converges by direct comparison test
(f) diverges since it is a harmonic series (g) converges by alternating series theorem
(h) converges since it is a telescoping series (i) diverges by divergence test (j) converge by direct comparison test (k) converges by limit comparison and integral test

10. 0.948

11. (a) Yes (b) about 0.17 (c) $R_3 \le \frac{1}{64}$

12. (a) 0 (b) [-1,1) (c) ∞, sin x

13. (a) $\sum_{n=0}^{\infty} (-1)^n (\frac{x}{5})^{n+1}, |x| < 5$ (b) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{3n+2}}{3n+2} + c$

14. $e^{3/5} - 1$

15. (a) $x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - ... + (-1)^{n+1}\frac{1}{n}x^n + ... = \sum_{n=1}^{\infty} \frac{(-1)^{n+1} x^n}{n}$

(b) $\sum_{n=0}^{\infty} \frac{x^{n+1}}{n!}$ (c) $1 + \frac{1}{3}x + \sum_{n=2}^{\infty} \frac{(-1)^{n+1} 2 \cdot 5 \cdot 8 \cdots (3n-4) x^n}{3^n \cdot n!}, |x| < 1$

16. (a) a circle centered at (2,0) and radius 2 (b) a line y = 2x -4 (c) a line thru (0,0) and slope is $\frac{1}{\sqrt{3}}$

17. (a) $x = a\cos t, y = -b\sin t, 0 \le t \le 2\pi$ (b) $x = a\cos 2t, y = b\sin 2t, 0 \le t \le 2\pi$

18. $x = -1 + t,\ y = 1 + t,\ z = 3t$

19. $y + z = 1$

20. $f_x = \ln\frac{x+y}{x-y} - \frac{2xy}{x^2 - y^2}$, $f_y = \frac{2x^2}{x^2 - y^2}$