Math 192 Review2 Name:

This exam is based on sections 5.5 thru 5.10 and no graphic calculator is allowed on exams. It is only a review and you also should go over other materials carefully for the test.

1. Evaluate the following. DON't use the table for these problems! If necessary, use the Comparison theorem and if needed, use $\frac{d}{dx}\csc x = -\csc x \cot x$ & $\int \csc x dx = \ln|\csc x - \cot x| + c$

(a) $\int_0^{\pi/4} \sin^2 x \cos x \, dx$ (b) $\int_1^4 \frac{1}{(1+\sqrt{x})^2} \frac{1}{\sqrt{x}} \, dx$ (c) $\int_{-3}^3 \frac{x}{\sqrt{1+3x^2}} \, dx$ (d) $\int_e^{e^4} \frac{dx}{x\sqrt{\ln x}}$

(e) $\int_{-1}^2 \frac{x}{\sqrt{x+2}} \, dx$ (f) $\int_1^e \ln x \, dx$ (g) $\int_0^\pi x \cos x \, dx$ (h) $\int_1^4 \frac{\ln x}{x^2} \, dx$ (i) $\int \cos(\ln x) \, dx$

(j) $\int_0^{\pi/12} \sin^2 u \, du$ (k) $\int_1^2 \sqrt{u^2-1} \, du$ (l) $\int \frac{1}{x^2-4} \, dx$ (m) $\int \frac{7x-23}{x^2-7x+12} \, dx$

(n) $\int_{-1}^1 \frac{\sin x}{1+x^2+x^4} dx$ (o) $\int_1^\infty x^{-2} \, dx$ (p) $\int_{-\infty}^\infty xe^{-x^2} \, dx$ (q) $\int_0^1 \frac{\ln x}{x^2} dx$ (r) $\int_3^\infty \frac{dx}{x^2-x-2}$

(s) $\int_1^\infty \frac{\cos^2 x}{x^3} dx$ (t) $\int_0^\infty \frac{1}{\sqrt{x}+1} dx$ (u) $\int \frac{1}{t^3\sqrt{t^2-1}} \, dt$

2. Use the Trapezoid Rule and Midpoint Rule with n = 4 to approximate the integral $\int_1^5 \frac{1}{x} \, dx$.

3. Estimate $\int_1^3 \frac{1}{x^2} \, dx$ using the Trapezoid Rule with n = 4. Then use the error bound $|E_T| \le \frac{K(b-a)^3}{12n^2}$ to estimate the accuracy.

4. Derive the following formula using the trigonometric substitution.

(a) $\int \frac{\sqrt{a^2+u^2}}{u} du = \sqrt{a^2+u^2} + a \ln \left| \frac{\sqrt{a^2+u^2}-a}{u} \right| + c$

(b) $\int \frac{\sqrt{a^2-u^2}}{u^2} du = -\frac{1}{u}\sqrt{a^2-u^2} - \sin^{-1}\frac{u}{a} + c$

5. Write the definition of the improper integrals $\int_a^\infty f(x)dx$, $\int_{-\infty}^b f(x)dx$ and $\int_{-\infty}^\infty f(x)dx$.

6. Use the table of integrals on the reference page to evaluate the integral.

(a) $\int \sqrt{x^2+x+1} dx$ (b) $\int \frac{\cot x}{\sqrt{1+2\sin x}} dx$ (c) $\int e^x \sqrt{1-e^{2x}} dx$

7. Find the partial fraction decomposition of the function below. Determine the coefficients as well.

(a) $\frac{1}{x(2x^2+x+2)}$ (b) $\frac{3x^2-x+3}{(2x+1)(x^2+4)}$

Answer:

1.(a) $\sqrt{2}/12$ (b) 1/3 (c) 0 (d) 2 (e) 2/3 (f) 1 (g) -2 (h) ¾-ln2/2 (i) $\frac{x}{2}[\cos(\ln x)+\sin(\ln x)]+c$

(j) $\frac{\pi-3}{24}$ (k) $\sqrt{3}-1/2\ln(2+\sqrt{3})$ (l) $\frac{1}{4}\ln|\frac{x-2}{x+2}|+c$ (m) $\ln((x-3)^2|x-4|^5)+c$

(n) 0 (o) 1 (p) 0 (q) Divergent (r) 1/3ln4 (s) Convergent (t) Divergent

(u) $\frac{1}{2}\cos^{-1}\left(\frac{1}{t}\right)+\frac{1}{2}\frac{\sqrt{t^2-1}}{t^2}+c$

2. $101/60, 496/315$

3. $|E_T|\leq\frac{1}{4}$

4. Please do them carefully for practices.

5. Check out the class note or textbook

6. (a) $\frac{2x+1}{4}\sqrt{x^2+x+1}+\frac{3}{8}\ln(x+\frac{1}{2}+\sqrt{x^2+x+1})+c$

(b) $\ln|\frac{\sqrt{1+2\sin x}-1}{\sqrt{1+2\sin x}+1}|+c$ (c) $\frac{1}{2}[e^x\sqrt{1-e^{2x}}+\sin^{-1}(e^x)]+c$

7. (a) $\frac{1}{2x}-\frac{x+1/2}{2x^2+x+2}$ (b) $\frac{1}{2x+1}+\frac{x-1}{x^2+4}$

List of topics

Sec 5.5: The Substitution Rule
Substitution rule; Integrals of Symmetric Functions (Odd, or Even Functions)

Sec 5.6: Integration by Parts
Integration by Parts; Tabular Integration by Parts; Substitution & Integration by Parts Combined

Sec 5.7: Additional Techniques of Integration
Some integrals requiring Trig Identities; Trig Substitution; Partial Fractions (Several Cases)

Sec 5.8: Integration using Tables
Evaluate integrals using tables

Sec 5.9: Approximate Integration
Review of Riemann Sum, LHS, RHS, Midpoint Rule, Trapezoid Rule, Error in M_n and T_n

Sec 5.10: Improper Integrals
Type 1 Improper Integrals (infinite intervals); Convergent and Divergent Improper Integrals;
Type 2 Improper Integrals (discontinuous integrands): Comparison Theorem