Review 3 (Answer Key)

1. Find the limit of the following sequence, if it converges. If it diverges, write *DIV* for your answer. Write the exact answer. Do not round.

$$a_n = \frac{2n+7}{5n-6}$$

Answer: ________________

2. Find the limit of the following sequence, if it converges. If it diverges, write *DIV* for your answer. Write the exact answer. Do not round.

$$a_n = \sin^{-1}\left(\frac{5n^2+n+1}{5n^3+4}\right)$$

Answer: ________________

3. Determine the first five terms of the sequence whose n^th term is defined as follows. Please enter the five terms in the boxes provided in sequential order. Please simplify your solution.

$$a_1 = 1,\ a_2 = 3, \text{ and } a_n = -5a_{n-1} + 3a_{n-2} \text{ for } n \geq 3$$

Answer: ________________, ________________, ________________, ________________, ________________

4. Decide whether the series $\sum_{n=0}^{\infty} \frac{7^{n-1}}{9^n}$ converges or diverges. If the series converges, find its sum. If the series diverges, explain why.

A) The series converges to $\frac{9}{112}$.
B) The series diverges because $\lim\limits_{n\to\infty} a_n \neq 0$.
C) The series diverges because $\lim\limits_{n\to\infty} s_n$ does not exist.
D) The series diverges because $\lim\limits_{n\to\infty} s_n = \infty$.
E) The series converges to $\frac{9}{14}$.

5. Find all values of x for which the geometric series $\sum_{n=1}^{\infty} 4(3-6x)^{n-1}$ converges.

A) $x > \frac{2}{3}$ and $x < \frac{1}{3}$
B) $x > \frac{1}{3}$
C) $\frac{1}{3} < x < \frac{2}{3}$
D) $\frac{11}{24} < x < \frac{13}{24}$
E) $-\frac{2}{3} < x < -\frac{1}{3}$

6. Decide whether the series $\sum_{n=0}^{\infty} \frac{4+7^n-9^n}{13^n}$ converges or diverges. If the series converges, find its sum. If the series diverges, explain why.

A) The series diverges because $\lim\limits_{n\to\infty} s_n = \infty$.
B) The series converges to $\frac{5811}{1540}$.
C) The series diverges because $\lim\limits_{n\to\infty} a_n \neq 0$.
D) The series converges to $\frac{13}{4}$.
E) The series diverges because $\lim\limits_{n\to\infty} s_n$ does not exist.

7. Determine whether the geometric series $13+\frac{13}{5}+\frac{13}{25}+\frac{13}{125}+\frac{13}{625}+\cdots$ converges or diverges. If the series converges, find its sum. Write the exact answer. Do not round. If the series diverges, write *DIV* for your answer.

Answer: _______________

8. Use the Integral Test to determine whether the series $\sum_{n=1}^{\infty}\frac{6}{n^{\frac{6}{5}}}$ converges or diverges.

A) Converges; $\int_1^{\infty}\frac{6}{x^{\frac{6}{5}}}dx=-6$ B) Converges; $\int_1^{\infty}\frac{6}{x^{\frac{6}{5}}}dx=-30$

C) Diverges; $\int_1^{\infty}\frac{6}{x^{\frac{6}{5}}}dx=-\infty$ D) Converges; $\int_1^{\infty}\frac{6}{x^{\frac{6}{5}}}dx=30$

E) Diverges; $\int_1^{\infty}\frac{6}{x^{\frac{6}{5}}}dx$ cannot be determined F) Diverges; $\int_1^{\infty}\frac{6}{x^{\frac{6}{5}}}dx=\infty$

9. Determine whether the Integral Test can be applied to the series $\sum_{n=1}^{\infty}\frac{(-1)^n}{7n^4}$ and give a reason for your answer. Select all answers that apply.

A) The Integral Test can be applied.
B) The Integral Test cannot be applied.
C) The terms of the series are eventually positive.
D) The terms of the series are not eventually positive.
E) The terms of the series are eventually monotonically decreasing.
F) The terms of the series are not eventually monotonically decreasing.

10. Use the Integral Test to determine whether the series $\sum_{n=1}^{\infty}\frac{2}{(n+3)(n+4)^2}$ converges or diverges.

A) Converges; $\int_1^{\infty}\frac{2}{(x+3)(x+4)^2}dx=-2\ln(\frac{3}{4})+\frac{2}{5}$

B) Diverges; $\int_1^{\infty}\frac{2}{(x+3)(x+4)^2}dx=-\infty$

C) Converges; $\int_1^{\infty}\frac{2}{(x+3)(x+4)^2}dx=2\ln(\frac{4}{5})+\frac{2}{5}$

D) Converges; $\int_1^{\infty}\frac{2}{(x+3)(x+4)^2}dx=-2\ln(\frac{4}{5})-\frac{2}{5}$

E) Diverges; $\int_1^{\infty}\frac{2}{(x+3)(x+4)^2}dx=\infty$

F) Diverges; $\int_1^{\infty}\frac{2}{(x+3)(x+4)^2}dx$ cannot be determined

11. Find the smallest possible value of n to approximate the sum of the series $\sum_{n=1}^{\infty}19ne^{-n^2}$ within the error $\varepsilon=8\times10^{-4}$. Then provide the requested estimate of the sum of the series s. Round any intermediate calculations, if needed, to no less than six decimal places, and round the final estimated sum to five decimal places. (**Hint:** Consider the midpoint of the interval $\left[s_n+\int_{n+1}^{\infty}f(x)dx\,,s_n+\int_n^{\infty}f(x)dx\right]$ as your estimate for s.)

Answer: $n=$_______________

$s\approx$_______________

12. Use the Direct Comparison Test to determine whether the series $\sum_{n=0}^{\infty} \frac{3\sin^2 n}{8^n}$ converges or diverges.

A) Converges; compare with the series $\sum_{n=0}^{\infty} \frac{3}{n!}$
B) Converges; compare with the series $\sum_{n=0}^{\infty} \frac{3}{8^n}$
C) Diverges; compare with the series $\sum_{n=0}^{\infty} \frac{3}{8^n}$
D) Diverges; compare with the series $\sum_{n=0}^{\infty} \frac{3}{n!}$

13. Use any previously considered convergence test to determine whether the series $\sum_{n=0}^{\infty} \frac{1}{6^n+9}$ converges or diverges.

A) Diverges since $\sum_{n=0}^{\infty} \frac{1}{6^n}$ diverges
B) Converges since $\sum_{n=0}^{\infty} \frac{1}{6^n}$ converges
C) Converges since $\sum_{n=0}^{\infty} (6^n + 9)$ converges
D) Diverges since $\sum_{n=0}^{\infty} (6^n + 9)$ diverges

14. Use the Direct Comparison Test to determine whether the series $\sum_{n=2}^{\infty} \frac{2}{7n-\sqrt{n}}$ converges or diverges.

A) Converges; compare with the series $\sum_{n=2}^{\infty} \frac{2}{\sqrt{n}}$
B) Converges; compare with the series $\sum_{n=2}^{\infty} \frac{2}{7n}$
C) Diverges; compare with the series $\sum_{n=2}^{\infty} \frac{2}{\sqrt{n}}$
D) Diverges; compare with the series $\sum_{n=2}^{\infty} \frac{2}{7n}$

15. Use any previously considered convergence test to determine whether the series $\sum_{n=0}^{\infty} \frac{1}{2n^2-4}$ converges or diverges.

A) Diverges since $\sum_{n=0}^{\infty} (2^{n^2} - 4)$ diverges
B) Converges since $\sum_{n=0}^{\infty} (2^{n^2} - 4)$ converges
C) Converges since $\sum_{n=0}^{\infty} \frac{1}{2n^2}$ converges
D) Diverges since $\sum_{n=0}^{\infty} \frac{1}{2n^2}$ diverges

16. Use the Ratio Test to determine whether the series $\sum_{n=0}^{\infty} \frac{n^2+2}{(n+3)^2}$ converges or diverges. If the Ratio Test is inconclusive, determine the convergence or divergence of the series by some other means.

A) Inconclusive; $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = 1$ but diverges by another test
B) Diverges; $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \frac{32}{27}$
C) Converges; $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = 0$
D) Converges; $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \frac{27}{32}$
E) Diverges; $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \infty$
F) Inconclusive; $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = 1$ but converges by another test

17. Use the Root Test to determine whether the series $\sum_{n=1}^{\infty}(1+\frac{8}{3n})^{3n^2}$ converges or diverges. If the Root Test is inconclusive, determine the convergence or divergence of the series by some other means.

A) Inconclusive; $\lim_{n\to\infty} \sqrt[n]{a_n} = 1$ but converges by another test
B) Inconclusive; $\lim_{n\to\infty} \sqrt[n]{a_n} = 1$ but diverges by another test
C) Diverges; $\lim_{n\to\infty} \sqrt[n]{a_n} = e^8$
D) Diverges; $\lim_{n\to\infty} \sqrt[n]{a_n} = 8$
E) Converges; $\lim_{n\to\infty} \sqrt[n]{a_n} = \frac{1}{e^8}$
F) Converges; $\lim_{n\to\infty} \sqrt[n]{a_n} = 0$

18. Use the Ratio Test to determine whether the series $\sum_{n=1}^{\infty}\frac{n^2}{(6e)^n}$ converges or diverges. If the Ratio Test is inconclusive, determine the convergence or divergence of the series by some other means.

A) Inconclusive; $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = 1$ but converges by another test
B) Converges; $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \frac{1}{6e}$
C) Diverges; $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \infty$
D) Converges; $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = 0$
E) Inconclusive; $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = 1$ but diverges by another test
F) Diverges; $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = 6e$

19. Use the Ratio Test to determine whether the series $\sum_{n=1}^{\infty}\frac{4^n}{(n+9)n}$ converges or diverges. If the Ratio Test is inconclusive, determine the convergence or divergence of the series by some other means.

A) Inconclusive; $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = 1$ but diverges by another test
B) Converges; $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = 0$
C) Converges; $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \frac{1}{4}$
D) Inconclusive; $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = 1$ but converges by another test
E) Diverges; $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \infty$
F) Diverges; $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = 4$

20. Determine whether the series $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{4}{5n^2+20}$ converges absolutely, converges conditionally, or diverges.

A) Converges absolutely
B) Converges conditionally
C) Diverges

21. Determine whether the alternating series $\sum_{n=0}^{\infty}(-1)^n \frac{n^2+5}{n^2+8}$ converges and give a reason for your answer. Select all of the correct statements that apply to the given series.

A) The series converges.
B) The series diverges.
C) The sequence $\{a_n\}$ is eventually monotonically decreasing.
D) The sequence $\{a_n\}$ is not eventually monotonically decreasing.
E) The sequence $\{a_n\}$ is eventually all positive terms.
F) The sequence $\{a_n\}$ is not eventually all positive terms.
G) $\lim_{n\to\infty} a_n > 0$
H) $\lim_{n\to\infty} a_n < 0$
I) $\lim_{n\to\infty} a_n = 0$

1. Correct Answer: $\frac{2}{5}$

2. Correct Answer: 0

3. Correct Answer: $1, 3, -12, 69, -381$

4. Correct Answer: The series converges to $\frac{9}{14}$.

5. Correct Answer: $\frac{1}{3} < x < \frac{2}{3}$

6. Correct Answer: The series converges to $\frac{13}{4}$.

7. Correct Answer: $\frac{65}{4}$

8. Correct Answer: Converges; $\int_1^\infty \frac{6}{x^{\frac{6}{5}}} dx = 30$

9. Correct Answer: The Integral Test cannot be applied., The terms of the series are not eventually positive., The terms of the series are not eventually monotonically decreasing.

10. Correct Answer: Converges; $\int_1^\infty \frac{2}{(x+3)(x+4)^2} dx = -2\ln(\frac{4}{5}) - \frac{2}{5}$

11. Correct Answer: $n = 3 \;\; s \approx 7.69333$

12. Correct Answer: Converges; compare with the series $\sum_{n=0}^{\infty} \frac{3}{8^n}$

13. Correct Answer: Converges since $\sum_{n=0}^{\infty} \frac{1}{6^n}$ converges

14. Correct Answer: Diverges; compare with the series $\sum_{n=2}^{\infty} \frac{2}{7n}$

15. Correct Answer: Converges since $\sum_{n=0}^{\infty} \frac{1}{2n^2}$ converges

16. Correct Answer: Inconclusive; $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = 1$ but diverges by another test

17. Correct Answer: Diverges; $\lim_{n\to\infty} \sqrt[n]{a_n} = e^8$

18. Correct Answer: Converges; $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \frac{1}{6e}$

19. Correct Answer: Diverges; $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = 4$

20. Correct Answer: Converges absolutely

21. Correct Answer: The series diverges.,The sequence $\{a_n\}$ is not eventually monotonically decreasing.,The sequence $\{a_n\}$ is eventually all positive terms., $\lim_{n\to\infty} a_n > 0$