Math 192 Review 4 Name:

Exam 4 is based upon the sections 7.1-7.4 and 8.1-8.3(part).

I may include some problems in the exam asking you to prove that we have done in class.

1. Test the following series for convergence. State the method used clearly and find the sum if possible. Try the following: limit of n-th partial sum (definition), div test, integral test, DCT.

(a) $\sum_{n=1}^{\infty} \ln(\frac{n}{n+1})$ (b) $\sum_{n=1}^{\infty} \frac{10^{n-1}+3}{2^n}$ (c) $\sum_{n=1}^{\infty} \frac{n}{n+1}$ (d) $\sum_{n=1}^{\infty} \frac{1}{(2n-1)(2n+1)}$ (e) $\sum_{n=1}^{\infty} \frac{\sqrt{n+1}}{n^2+2}$

(f) $\sum_{n=2}^{\infty} \frac{1}{n \ln n^3}$ (g) $\sum_{n=1}^{\infty} \frac{\sin^2 n}{10^n}$ (h) $\sum_{n=1}^{\infty} \frac{1+\cos 3n}{1+(1.2)^n}$ (i) $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$

2. Find the values of x for which the series converges. If possible, find the sum, too.

(a) $1-\frac{1}{2}(x-2)+\frac{1}{4}(x-2)^2-\frac{1}{8}(x-2)^3+...+(-\frac{1}{2})^n(x-2)^n+...$ (b) $\sum_{n=1}^{\infty} (\ln x)^n$

3. A ball is dropped from a height of 8 feet. Each time it strikes the ground after falling from a height of t feet it bounds to a height of ¾ t feet. Find the total distance traveled by the ball.

4. Consider the following sequence and determine whether it converges or diverges. If it converges, find its limit.

(a) $a_n = \left(\frac{-3}{4}\right)^n$ (b) $b_n = \frac{\ln n}{n}$ (c) $a_n = \frac{n \cos n}{n^2+1}$ (d) $a_n = \frac{3n+4}{2n-4}$ (e) $a_n = \frac{7 \cdot 4^n}{3^n}$

(f) $a_n = \sin(\frac{n\pi}{2})$ (g) $a_n = \frac{\sqrt{n+1}}{5n+3}$ (h) $a_n = \frac{5\cos n + n}{n^2}$ (i) $a_n = \frac{2^n}{n!}$ (j) $a_n = \frac{\sqrt{n+1}-\sqrt{n}}{\sqrt{n+1}+\sqrt{n}}$

(k) $a_n = \frac{2n^3+3n-3}{n(n+1)(n+2)}$ (l) $a_n = \sqrt[n]{n}+\sqrt[n]{10}$ (m) $a_n = (1+\frac{2}{n})^n$ (You must show all necessary work to find the limit value of the sequences in (l) and (m))

5. Consider the recursive sequence given by $x_1 = 1; x_{n+1} = \frac{x_n^{\,2}+2}{2x_n}, n \ge 1$. Suppose it is convergent and find the limit.

6. Find the formula for the nth term of the sequence $\frac{1}{4}, -\frac{2}{9}, \frac{3}{16}, -\frac{4}{25}, ...$ and find the limit value.

7. For the sequence $\{b_n\} = \left\{\frac{1}{2^{n-1}}\right\}_{n=1}^{\infty}$, answer the following questions.

(a) Find the limit value.

(b) Find a positive integer N such that $|b_n - 0| < 0.0003$ for $n > N$.

8. If the nth partial sum of a series $\sum_{n=1}^{\infty} a_n$ is $s_n = \frac{n}{n+1}$, find a_n, $\sum_{n=1}^{\infty} a_n$ and $\lim_{n\to\infty} a_n$.

9. (True or False) Justify your answer. No points will be given without a valid reasoning.

(a) If $\lim_{n\to\infty} a_n = 0$, then $\sum_{n=1}^{\infty} a_n$ converges.

(b) The series $\sum_{n=1}^{\infty} n^{-\sqrt{2}}$ converges.

(c) If $\lim_{n\to\infty} a_n = L$, then $\lim_{n\to\infty} a_{2n+1} = L$.

10. Solve the differential equation or IVP.

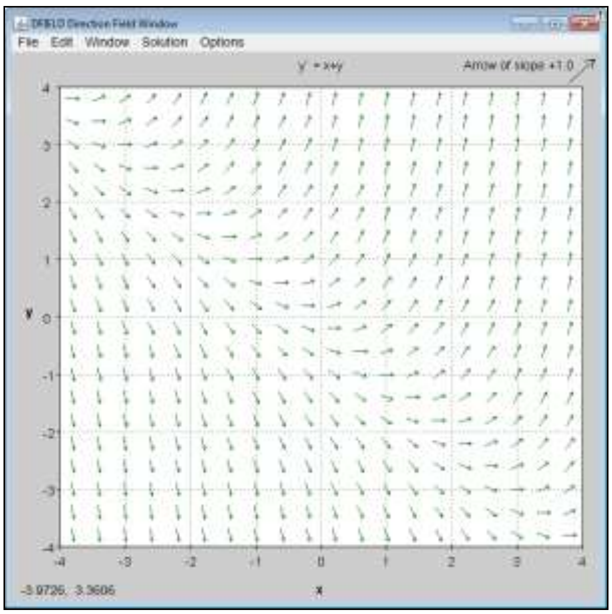
(a) $y' = y^2$ (b) $y' = \dfrac{\ln x}{xy}, y(1) = 2$

11. A tank contains 500 liters of brine with 10kg of dissolved salt. Brine that contains 0.03 kg of salt per liter of water enters the tank at a rate of 10 liters per minutes. The well-mixed solution drains from the tank at the same rate. How much salt remains in the tank after half an hour?

12. A cup of hot chocolate has temperature $80^\circ C$ in a room kept at $20^\circ C$. After half an hour the hot chocolate cools to $60^\circ C$. When will the chocolate have cooled to $40^\circ C$?

13. A direction field is given below. Which of the following represents its differential equation? Justify your answer and sketch the integral curve passing thru the origin.

(a) $y' = \sin x$ (b) $y' = -y$ (c) $y' = x + y$ (d) $y' = x^2$ (e) $y' = y^2$



14. For what constant values of k does the function $y = \cos kt$ satisfy the differential equation $4y'' = -25y$?

Answer key

1. (a) div by definition (b) div by geometric series or div test (c) div by div test
(d) con to 0.5 by telescoping series (e) con by DCT (f) div by integral test
(g), (h) con by DCT (i) con by integral test
(i) con to 0.5 by telescoping series

2. (a) (0,4) $S = \frac{1}{1+0.5(x-2)} = \frac{2}{x}$ (b) $\frac{1}{e} < x < e, s = \frac{\ln x}{1-\ln x}$

3. 56ft

4. (a), (b), (c), (g), (h), (i), (j) converge to zero, (d) conv to 3/2, (k), (l) conv to 2 (m) conv to e^2
(e),(f) diverge

5. $\sqrt{2}$

6. $a_n = \frac{(-1)^{n-1} n}{(n+1)^2}$ converges to zero by Squeeze theorem. Show the details.

7. (a) 0 (b) any integer bigger than 13

8. $a_n = \frac{1}{n(n+1)}, \sum_{n=1}^{\infty} a_n = 1, \lim_{n\to\infty} a_n = 0$

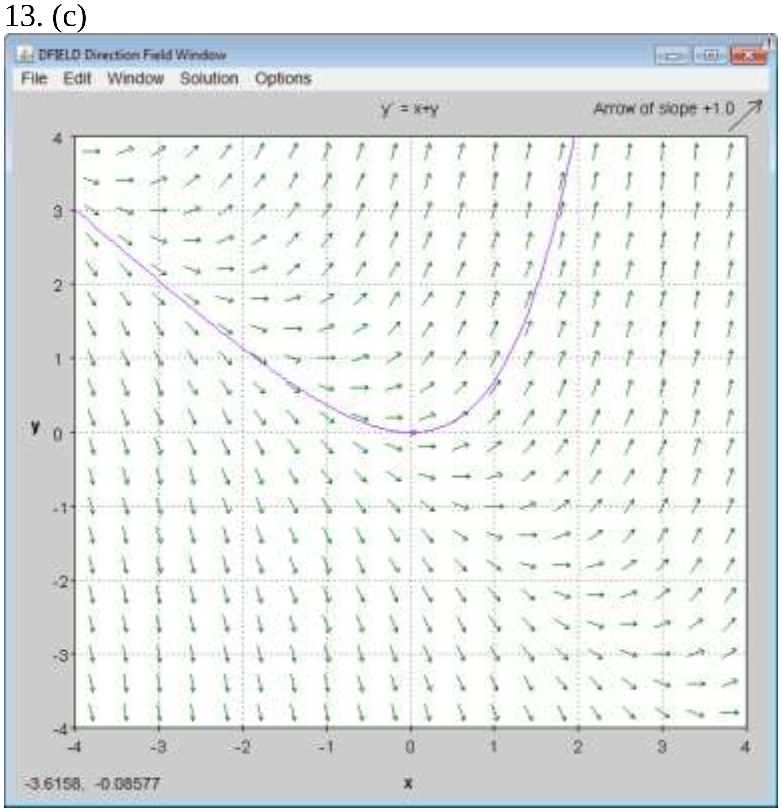
9. (a) F (b) T (c) T

10. (a) $y = -\frac{1}{x+c}$ (b) $y = \sqrt{4+(\ln x)^2}$

11. 12.26 kg

12. 1 hour 21 minutes

13. (c)



14. $k = \pm 2.5$