Math 192 Review 5 Name:

Exam 5 is based upon the sections 8.3-8.8. You should go over all the quiz, worksheet, notebook and homework problems with this review for the test. I may include a problem to ask you to explain some concepts and formulas discussed in class.

1. (a) Use series to compute $\int_0^1 x\cos x\, dx$ correct to three decimal places.

(b) Use integration by parts to compute $\int_0^1 x\cos x\, dx$.

(c) Compare your answers in parts (a) and (b) above.

2. Find the Taylor series generated by f at a.

(a) $f(x)=\sqrt{x}$ at $a=9$.

(b) $f(x)=\sin \pi x$ at zero

3. Find the Maclaurin series for each of the following.

Note: (1) Find the first four nonzero terms of the series and then, use the summation notation.

(2) You are free to use the known expression for $\frac{1}{1-x}$, e^x, $\cos x$, $\sin x$ if needed.

(a) $\frac{1}{1+x^2}$ (b) $\tan^{-1} x$ (c) $\frac{1}{(1+x)^2}$ (d) xe^{-x}

4. Evaluate the following.

(a) $\sum_{n=0}^{\infty}\frac{(-1)^n \pi^{2n+1}}{3^{2n}(2n+1)!}$ (b) $\sum_{n=1}^{\infty}\frac{(-1)^n}{n!}$ (c) $\int_0^1 e^{-x^2}dx$ accurate to two decimal places (d) $\lim_{x\to 0}\frac{\cos x-1}{x^2}$

5. What is the smallest value of n that will guarantee (according to Taylor's formula) that the Taylor polynomial T_n at the number 0 will be within 0.0001 of e^x for $0\le x\le 1$?

6. Find the Taylor polynomial T_3 for the function $f(x)=\frac{5x}{2+4x}$ at the point 0.

7. Test the following series for convergence, absolute convergence, conditional convergence State the method used clearly.

(a) $\sum_{n=1}^{\infty}\frac{n\ln n}{3^n}$ (b) $\sum_{n=0}^{\infty}\frac{\sqrt{n+1}}{n^2+2}$ (c) $\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{\sqrt{n+1}}$ (d) $\sum_{n=1}^{\infty}\ln(1+\frac{1}{n})$ (e) $\sum_{n=2}^{\infty}\frac{1}{n\ln n^3}$ (f) $\sum_{n=1}^{\infty}\frac{\sin^2 n}{10^n}$

(g) $\sum_{n=1}^{\infty}\frac{1}{1+\ln n}$ (h) $\sum_{n=1}^{\infty}\frac{10^{n-1}+3}{2^n}$ (i) $\sum_{n=1}^{\infty}\frac{n}{n+1}$ (j) $\sum_{n=1}^{\infty}\frac{n^2+1}{n^3+1}$ (k) $\sum_{n=1}^{\infty}\frac{1+\cos 3n}{1+(1.2)^n}$

(l) $\sum_{n=1}^{\infty}\frac{1}{(2n-1)(2n+1)}$ (m) $\sum_{n=2}^{\infty}\frac{1}{n(\ln n)^2}$ (n) $\sum_{n=1}^{\infty}\frac{1}{\sqrt{n^2+1}}$

8. Which of the following series are convergent, but not absolutely convergent?

(1) $\sum_{n=1}^{\infty}(-e)^{-n}$ (2) $\sum_{n=1}^{\infty}(-1)^n n^{-1}$ (3) $\sum_{n=1}^{\infty}(-1)^n n^{-2}$

9. Find the interval and radius of convergence of the series $\sum_{n=1}^{\infty}\frac{(-1)^n(x-3)^n}{5^n(n+1)}$.

10. (a) Show that $\sum_{n=0}^{\infty} \frac{x^n}{n!}$ converges for every x. (b) Deduce that $\lim_{n\to\infty} \frac{x^n}{n!} = 0$.
(c) What is the function that has this power series representation?

11. Use the binomial series to find the Maclaurin series for the following. State the radius of convergence.

(a) $\sqrt{1+x}$ (b) $\sqrt{4+x}$ (c) $\frac{1}{(1+x)^4}$ (d) $\frac{x^2}{\sqrt{1-x^3}}$

Answer Key

1. (a) 0.382 (b) cos1+sin1-1 (c) 0.38177

2. (a) $3+\frac{1}{6}(x-9)-\frac{1}{216}(x-9)^2+\frac{1}{3888}(x-9)^3+...$ (b) $\sum_{n=0}^{\infty}(-1)^n \frac{(\pi x)^{2n+1}}{(2n+1)!}$

3. (a) $\sum_{n=0}^{\infty}(-1)^n x^{2n}$ (b) $\sum_{n=0}^{\infty}(-1)^n \frac{1}{2n+1} x^{2n+1}$ (c) $\sum_{n=1}^{\infty}(-1)^{n+1} n x^{n-1}$ (d) $\sum_{n=0}^{\infty}(-1)^n \frac{x^{n+1}}{n!}$

4. (a) $\frac{3\sqrt{3}}{2}$ (b) $e^{-1}-1$ (c) 0.747486772 (d) -1/2

5. 7

6. $5/2x-5x^2+10x^3$

7. (a) con by Ratio test (b) con. by DCT or LCT (c) cond. converge (d) diverges by definition (e) div by integral test (f) con by DCT (g) div by LCT (h) div by geometric series (i) div by nth term test (j) div by LCT (k) con by DCT (l) con to 0.5 (m) conv by integral test (n) div by LCT

8. (2) only

9. (-2,8] r = 5

10. (a) Ratio test (b) Find the reason why! (c) e^x

11. Note: Here (2n-1)!!=$1\cdot 3\cdot 5\cdot 7\cdots(2n-3)\cdot(2n-1)$ means the product of odd numbers.

(a) $1+\frac{1}{2}x+\sum_{n=2}^{\infty}\frac{(-1)^{n-1}}{n!2^n}(2n-3)!!x^n$, $|x|<1, R=1$

(b) $2[1+\frac{x}{8}+\sum_{n=2}^{\infty}\frac{(-1)^{n-1}}{n!2^{3n}}(2n-3)!!x^n, |x|<4, R=4$

(c) $1-4x+\sum_{n=2}^{\infty}\frac{(-1)^n 4\cdot 5\cdots(n+3)}{n!}x^n, |x|<1, R=1$

(d) $x^2[1+\frac{1}{2}x^3+\sum_{n=2}^{\infty}\frac{(2n-1)!!}{2^n n!}x^{3n}], |x|<1, R=1$