

1. Determine whether the following series is convergent or divergent and find the sum if possible.
If series is a geometric series, then state so.

(a) $\frac{2}{3} + \frac{2}{3^2} + \frac{2}{3^3} + \dots = \sum_{n=1}^{\infty} \frac{2}{3^n}$

(b) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{2^n}{3^n}$

(c) $0.9 + 0.09 + 0.009 + 0.0009 + \dots$

(d) $\sum_{n=1}^{\infty} \frac{1}{e^{2n}}$

(e) $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \dots$

(f) $-\frac{81}{100} + \frac{9}{10} - 1 + \frac{10}{9} - \dots$

(g) $\sum_{n=1}^{\infty} \frac{1}{n(n+2)}$

(h) $\sum_{n=1}^{\infty} \frac{3 + (-1)^n}{3^n}$

(i) $\sum_{n=0}^{\infty} \cos\left(\frac{\pi}{2n^2 - 1}\right)$

(j) $\sum_{n=1}^{\infty} \frac{1}{2 + 3^{-n}}$

2. Express the number $0.\overline{215}$ as a ratio of integers.

3. A series $\sum_{k=1}^{\infty} a_k$ has partial sums, s_n , given by $s_n = \frac{7n-2}{n}$.

(a) Is $\sum_{k=1}^{\infty} a_k$ convergent? If it is, find the sum.

(b) $\lim_{n \rightarrow \infty} a_n$

(c) Find $\sum_{k=1}^{200} a_k$.

4. True or False. Justify your answers. If true, prove it or false, give a counter example.

(a) If the series $\sum_{n=1}^{\infty} a_n$ converges, then $\lim_{n \rightarrow \infty} a_n = 0$.

(b) An infinite series $\sum_{n=1}^{\infty} a_n$ converges if the limit of the sequence of partial sums converges.

(c) There exists a convergent series $\sum_{n=1}^{\infty} a_n$ which satisfies $\lim_{n \rightarrow \infty} a_n \neq 0$.

(d) Let $a_n = \frac{n}{2n+1}$. This sequence a_n converges but the series $\sum_{n=1}^{\infty} a_n$ diverges.

(e) There is no divergent series $\sum_{n=1}^{\infty} a_n$ which satisfies $\lim_{n \rightarrow \infty} a_n = 0$.