Math 215 Review for Exam 1

This exam covers from Sec 2.1 thru 3.3 and you should review all the related materials for the exam. You are not allowed to use a graphic calculator for the exam.

1. (a) Express $A=\begin{pmatrix}1&2&0\\0&1&3\\3&8&7\end{pmatrix}$ as a product of elementary matrices. Is it invertible? Find its inverse.

(b) Is the system $A\vec{x}=\vec{b},\ \vec{b}=\begin{pmatrix}-1\\-1\\-5\end{pmatrix}$ consistent? If so, how many are the solutions?

2. The following is the reduced row echelon form of the system $A\vec{x}=\vec{b}$. In each case, determine
(i) whether the original equations have a solution
(ii) if they do have a solution, whether or not it is unique
(iii) if it is not unique, on how many free parameters there are in the solution. Then write the solution explicitly.

(a) $\left[\begin{array}{ccccc|c}1&5&0&1&0&2\\0&0&1&1&0&-1\\0&0&0&0&0&2\end{array}\right]$ (b) $\left[\begin{array}{ccc|c}1&0&0&2\\0&1&0&-1\\0&0&1&2\end{array}\right]$ (c) $\left[\begin{array}{cccccc|c}1&1&-1&0&0&-2&1\\0&0&0&1&0&0&2\\0&0&0&0&1&0&2\\0&0&0&0&0&0&0\end{array}\right]$

3. Determine whether the set $\{\begin{pmatrix}1&2\\0&0\end{pmatrix},\begin{pmatrix}0&0\\2&1\end{pmatrix},\begin{pmatrix}0&2\\0&1\end{pmatrix}\}$ is linearly independent or not.

Use the definition of being linearly independent.

4. Find the span$\{\begin{pmatrix}1\\-1\\2\end{pmatrix},\begin{pmatrix}2\\-3\\0\end{pmatrix},\begin{pmatrix}0\\0\\0\end{pmatrix}\}$.

5. If $A^{-1}=\begin{pmatrix}-2&4\\5&7\end{pmatrix},\ B^{-1}=\begin{pmatrix}1&-1\\2&3\end{pmatrix}$, find the following.

(a) A (b) $(AB)^{-1}$ (c) $(A^TB)^{-1}$

6. Compute the product of the partitioned matrix using block multiplication.

$$\left(\begin{array}{cc}2&0\\\hline 3&1\\-1&5\\1&2\end{array}\right)\left(\begin{array}{cc|cc}-1&2&3&0\\2&2&-1&2\end{array}\right)$$

7. Suppose that A and B are matrices. True or False. Justify your answer.

(a) $(A+B)^T=A^T+B^T$

(b) $(A+B)^{-1}=A^{-1}+B^{-1}$

(c) $(A+B)(A-B)=A^2-B^2$

(d) Every homogeneous system is consistent.

Answer key to review 1

1. (a) A= $\begin{pmatrix}1&0&0\\0&1&0\\3&0&1\end{pmatrix}\begin{pmatrix}1&0&0\\0&1&0\\0&2&1\end{pmatrix}\begin{pmatrix}1&0&0\\0&1&3\\0&0&1\end{pmatrix}\begin{pmatrix}1&2&0\\0&1&0\\0&0&1\end{pmatrix}$ and $A^{-1}=\begin{pmatrix}-17&-14&6\\9&7&-3\\-3&-2&1\end{pmatrix}$

(b) Yes, it has a unique solution (1,-1,0)

2. (a) no solution

(b) unique solution (2,-1,2)

(c) infinitely many solution

$$\begin{pmatrix}x_1\\x_2\\x_3\\x_4\\x_5\\x_6\end{pmatrix}=\begin{pmatrix}1+2t+u-s\\s\\u\\2\\2\\t\end{pmatrix}=\begin{pmatrix}1\\0\\0\\2\\2\\0\end{pmatrix}+t\begin{pmatrix}2\\0\\0\\0\\0\\1\end{pmatrix}+u\begin{pmatrix}1\\0\\1\\0\\0\\0\end{pmatrix}+s\begin{pmatrix}-1\\1\\0\\0\\0\\0\end{pmatrix}$$

3. They are linearly independent.

4. They span a plane $6x+4y-z=0$.

5. Use the facts we have done in class.

(a) $A=\begin{pmatrix}-7/34&4/34\\5/34&2/34\end{pmatrix}$ (b) $B^{-1}A^{-1}=\begin{pmatrix}-7&-3\\11&29\end{pmatrix}$ (c) $\begin{pmatrix}-6&-2\\8&31\end{pmatrix}$

6. $\left(\begin{array}{cc|cc}-2&4&6&0\\-1&8&8&2\\11&8&-8&10\\3&6&1&4\end{array}\right)$

7. (a) T (b) F (c) F (d) T

This is a brief outline of the main topics we had in class.

Chapter 2. Systems of Linear Equations
2.1 Introduction to Systems of Linear Equations
Systems of Linear Equations
Augmented Matrix
Backward and Forward Substitution Method
Three Possible Cases of Solutions of System of Linear Equations

2.2 Direct Methods for Solving Linear Systems
Elementary Row Operations
Gaussian Elimination and Row Echelon Form
Row Equivalent Matrices
Leading variables and Free variables
System that has infinitely many solutions
Rank of a Matrix
The Rank Theorem
Gaussian Jordan elimination and Reduced Row Echelon Form
Homogeneous Linear System

2.3 Spanning Sets and Linear Independence
Span and Spanning Set of Vectors
Linear Independence
Theorem 2.4, 2.5. 2.6 and 2.7.
(Please understand the meaning of these theorems and know how to apply them with examples.)

2.4 Applications
Network Analysis, Electrical Networks, Linear Economic Models

Chapter 3. Matrices
3.1 Matrix Operations
Terms of a Matrix, Row, Column, Diagonal Entries, Square Matrix, Identity Matrix
Two Matrices are Equal
Matrix Addition and Scalar Multiplication
Matrix Multiplication and Matrix Powers
Applications of Matrix Multiplications
Theorem 3.1 in page 144
Partitioned Matrices
The Transpose of a Matrix and its Properties
Symmetric Matrix

3.2 Matrix algebra
Algebraic properties of Matrix Addition and Scalar Multiplication
Linear Combinations and Span of matrices and Linear Independence
Properties of Matrix Multiplication and Properties of the Transpose

3.3 The Inverse of a Matrix
Definition of an Inverse of a Matrix
Theorems 3.6, 3.7, 3.8, 3.9, 3.10, 3.11, 3.12
Elementary Matrix and their Inverses
Express a Matrix as a Product of Elementary Matrices
Find the inverse of a 3 by 3 matrix