Math 215 Review 2

This exam covers from Sec 3.4 thru 4.3 and you should review all the related materials for the exam. You are not allowed to use a graphic calculator for the exam.

1. Show that the set $V = \{\begin{pmatrix} 4t \\ s+t \\ -3s+t \end{pmatrix} \in \mathbb{R}^3$: s and t are scalar} is a subspace of $\mathbb{R}^3$ and find a basis and dimension of V.

2. Let A=$\begin{pmatrix} 1 & 1 & -1 & -2 \\ -1 & -2 & 1 & 3 \\ 2 & 3 & 1 & 4 \end{pmatrix}$. Find a basis for the column space of A and a basis for the null space of A and verify the rank theorem.

3. Compute the determinant of $\begin{pmatrix} 1 & 1 & x \\ -1 & 7 & y \\ 2 & 8 & z \end{pmatrix}$ by cofactor expansion along the **third column**.

4. Use Cramer's Rule (no credit for other methods!) to solve the following system of linear equations.

$$\boxed{\begin{cases} x_1 + 2x_2 + 3x_3 = 2 \\ x_1 + x_3 = 3 \\ x_1 + x_2 - x_3 = 1 \end{cases}}$$

5. Show that $S = \{\begin{pmatrix} x \\ y \end{pmatrix} : xy = 0\}$ is not a subspace of $\mathbb{R}^2$.

6. Explain why $\{\begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}, \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}, \begin{pmatrix} 5 \\ -11 \\ 4 \end{pmatrix}\}$ is not a basis for $\mathbb{R}^3$.

7. Let $A = \begin{pmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{pmatrix}$. Show that detA=(b-a)(c-a)(c-b).

8. Determine whether the following is invertible and if so, find its inverse.

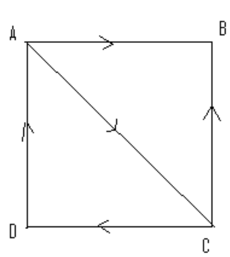
$$\boxed{\begin{pmatrix} 1 & 0 & 1 \\ 2 & -1 & 1 \\ 1 & 1 & 1 \end{pmatrix}}$$

9. Find the eigenvalues of the following matrix, and determine a basis for each eigenspace. A=$\begin{pmatrix} 0 & 4 \\ -1 & 5 \end{pmatrix}$.

10. Find an LU factorization of the following matrix $\begin{pmatrix} 2 & -1 & 1 \\ 4 & -1 & 4 \\ -2 & 1 & 2 \end{pmatrix}$ and use it to solve the system

$$\boxed{\begin{cases} 2x_1 - x_2 + x_3 = -1 \\ 4x_1 - x_2 + 4x_3 = -2 \\ -2x_1 + x_2 + 2x_3 = -2 \end{cases}}$$

11. (True or False) Justify your answer.
(a) There are some subspaces of $\mathbb{R}^n$ that do not have 0.
(b) The rank of any matrix equals to the dimension of its row space.
(c) If a matrix A has a zero as an eigenvalue, then A is not invertible.
(d) If a matrix A is row equivalent to a matrix B, then det A= det B.



12. Find a linear transformation $T:\mathbb{R}^2 \to \mathbb{R}^2$ such that $T\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ and $T\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 5 \end{bmatrix}$.

13. Determine the adjacency matrix of the digraph(left) and find the number of paths of length 2 between A and B.

Answer key to review 2

1. Show three things carefully: zero vector in it, closed under addition, closed under scalar multiplication. Basis $\{\begin{pmatrix} 0 \\ 1 \\ -3 \end{pmatrix}, \begin{pmatrix} 4 \\ 1 \\ 1 \end{pmatrix}\}$ and its dimension is 2.

2. Basis for column space $\begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$ and basis for null space is. $\begin{pmatrix} -2 \\ 1 \\ -3 \\ 1 \end{pmatrix}$

Rank theorem says for a m by n matrix A, n equals to the sum of rank of A and nullity of A which is in this case, 4=3+1.

3. $-22x-6y+8z$

4. (2.5,-1,0.5)

5-7, Please do it carefully.

8. It is invertible since det is not zero and its inverse is $\begin{pmatrix} -2 & 1 & 1 \\ -1 & 0 & 1 \\ 3 & -1 & -1 \end{pmatrix}$.

9. $\lambda = 1, \begin{pmatrix} 4 \\ 1 \end{pmatrix}; \lambda = 4, \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

10. $L = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix}, U = \begin{pmatrix} 2 & -1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 3 \end{pmatrix}$, (1,2,-1)

11. (a) F (b)T (c)T (d)F

12. $T\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x+y \\ 4x-y \end{pmatrix}$

13. A= $\begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix}$, and $A^2 = \begin{pmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{pmatrix}$ so that there exists only one path from A to B of length two.

This is a brief outline of the main topics we had in class.

Sec 3.4 The LU factorization
LU factorization, Solve the system using LU factorization

Sec 3.5 Subspaces, Basis, Dimension and Rank
definition of subspaces, row space, column space, null space of a matrix A, Basis, Nullity, Rank, The Basis theorem, The Rank theorem, The Fundamental theorem of invertible matrix II

Sec 3.6 Introduction to linear transformation
Definition of linear transformation, Matrix representation of a linear transformation (Theorem 3.30 and 3.31), The compose of two linear transformations, Homogeneous Coordinate system

Sec 3.7 Applications
Markov chain, Graph theory

Chapter 4. Eigenvalues and eigenvectors

4.1 Introduction
Definition of eigenvalues and eigenvectors, geometric meaning of eigenvalues and eigenvectors

4.2 Determinants
Determinant and inverse matrix of 2 by 2 matrix, Determinant of 3 by 3 matrix or higher, Laplace cofactor expansion theorem, Determinant of upper (lower) triangular matrix, Properties of determinant (Theorem 4.3, 4.6, 4.7. 4.8, 4.9, 4.10), The Cramer's Rule, Inverse matrix using adjoint

4.3 Eigenvalues and eigenvectors of n by n matrices
Find eigenvalue and eigenvector for 3 by 3 matrices, A square matrix A is invertible iff 0 is not an eigenvalue of A., the fundamental theorem of invertible matrix III, Theorem 4.18, 4.19 and 4.20