Math 240 Final Exam Name:

YOU MUST SHOW ALL YOUR WORK TO GET PROPER CREDITS.
NO PONTS WILL BE GIVEN ONLY FOR AN ANSWER.

1. Let S be a surface given by $x^2 + y^2 - z^2 = 1$, $z \geq 0$.
(a) Sketch the surface. Include several points on your sketch and identify the type of the surface.
(b) Find a parametrization of this surface.
(c) Sketch the several level curves for $z = k = 0, 1, 2$. Indicate the levels on your sketch.
(d) Find the equation of the tangent plane to the surface at the point $(0, \sqrt{5}, 2)$.

2. Consider the function $f(x, y) = \begin{cases} \dfrac{x^2 + 3y^2}{x - y} & \text{if } x \neq y \\ 0 & \text{if } x = y \end{cases}$

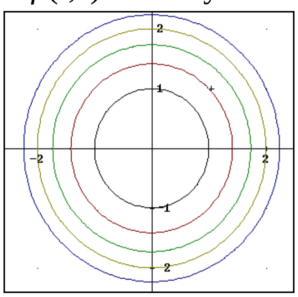
Compute $f_x(0,0)$ directly from the limit definition of a partial derivative.

3. Find the volume of the wedge cut from the cylinder $x^2 + y^2 = 1$ by the plane $z = -y$ and $z = 0$.

4. (a) State the Green's theorem.
(b) Using Green's theorem to find the work done by the force field $F(x, y) = x(x + y)\mathrm{i} + xy^2\mathrm{j}$ in moving a particle from the origin along the x-axis to (1,0), then along the line segment to (0,1), and then back to the origin along the y-axis.

5. Using the Divergence theorem to calculate the following surface integral $\iint_S F \bullet d\vec{S}$ i.e. flux of F across S, where $F(x, y, z) = (e^x \sin y)\mathrm{i} + (e^x \cos y)\mathrm{j} + (yz^2)\mathrm{k}$ and S is the surface of the box bounded by the planes $x = 0, x = 1, y = 0, y = 1, z = 0$ and $z = 2$.

6. Answer the following based on the level curves.
(a) Suppose that this is a contour map of a function $z = f(x, y) = y^2 + x^2$ and sketch the gradient $\nabla f(1,1)$ carefully on the sketch provided based upon the scale given.



(b) What is the directional derivative of f at (1,1) in the direction of v = <2,1>?
(c) In which direction does f increase most rapidly at (1,1)? Decrease most rapidly at (1,1)? Or change nothing at (1,1)?

7. Use **Stokes' theorem** to compute the integral $\iint_S \mathrm{curl} F \bullet d\vec{S}$, where $F(x, y, z) = xz\mathrm{i} + yz\mathrm{j} + xy\mathrm{k}$ and S is the part of the sphere $x^2 + y^2 + z^2 = 4$ that lies inside the cylinder $x^2 + y^2 = 1$ and above the xy-plane.

8. Evaluate the following integrals. You must sketch your domain (or region) of your integral.
(a) $\int_0^1 \int_0^{\sqrt{1-x^2}} e^{-(x^2+y^2)} dy dx$ (b) $\iint_R \sin(9x^2 + 4y^2) dA$ Consider an appropriate change of variables, where R is the region in the first quadrant bounded by the ellipse $9x^2 + 4y^2 = 1$.
(c) $\iiint_E \sqrt{x^2 + z^2} dV$, where E is the region bounded by the paraboloid $y = x^2 + z^2$ and the plane $y = 4$.

(d) $\int_C \nabla(x^3+3xy-2y)\, d\mathrm{r}$, where C is the line segment between (-1,0) and (1,1).

(e) $\int_C xds$, where C is the parabola $y=x^2$ between (-1,1) to (1,1).

(f) $\iint_S y\ dS$, where S is the surface $z=x+y^2,\ 0\le x\le 1,\ 0\le y\le 2$

9. The temperature at a point (x,y) is T(x,y), measured in degrees Celsius. A bug crawls so that its position after t seconds is given by $x=\sqrt{1+t}, y=2+\frac{1}{3}t,$ where x and y are measured in centimeters. The temperature function satisfies $T_x(2,3)=4$ and $T_y(2,3)=3$. How fast is the temperature rising on the bug's path after 3 seconds?

10. Use the Lagrange multipliers to find the extreme values of the function $f(x,y)=xy$ on the ellipse $\frac{x^2}{8}+\frac{y^2}{2}=1$.

Please go over all past exams, quizzes, worksheets and homework problems very thoroughly and carefully for the final exam. Final exam is at least 20% of the final grade so that it will affect your grade significantly.

Answers:

1. (a) Hyperboloid of one sheet (b) $r(x,y)=(x,y,\sqrt{x^2+y^2-1})$ (c) skip (d) $2z-\sqrt{5}y+1=0$
2. 1
3. 2/3
4. (b) -1/12
5. 2
6. (a) Be careful on the direction and the length of the vector! (b) $6/\sqrt{5}$ (c) The function f Increases most rapidly at (1,1) in the gradient vector direction <1,1>, decreases most rapidly in the opposite direction <-1,-1> and it does not change in any direction orthogonal to <1,1> which is either <1,-1> or <-1,1>.
7. 0
8. (a) $\frac{\pi(1-e^{-1})}{4}$ (b) $\frac{\pi}{24}(1-\cos 1)$ (c) $\frac{128\pi}{15}$ (d) 3 (e) 0 (f) $\frac{13\sqrt{2}}{3}$
9. $2^\circ C/\sec$
10. max 2 at (2,1) & (-2,-1) and min -2 at (-2,1) & (2,-1).