

Math 240 Key to review 2

1. (a) $x = 4 \cos t, y = 5 \sin t$ or $x = 4 \sin t, y = 5 \cos t, 0 \leq t \leq 2\pi$.

(b)

(Algebraically) Use parametrization $x = 4 \cos t, y = 5 \sin t$ to compute the curvature function. Then the

curvature function is $k(t) = \frac{20}{(\sqrt{16+9\cos^2 t})^3}$ (need some calculation). The curvature is minimum when the

denominator is maximum when $\cos^2 t = 1$ which means at $t = 0$ or $2\pi \Leftrightarrow (4,0), (-4,0)$. Its minimum curvature is $20/125=0.16$

Similarly, the curvature becomes maximum when $\cos^2 t = 0 \Leftrightarrow t = \pi/2, 3\pi/2 \Leftrightarrow (0,5), (0,-5)$ and the maximum curvature is 0.3125.

(Graphically) You can see the curvature is minimum at (4,0) and (-4,0) and becomes maximum at (0,5) and (0,-5).

2. (a) Sphere of radius 1 centered at the origin (b) paraboloid $z = x^2 + y^2$

3. $x = t, y = 2 + t, z = 1 + 2t$

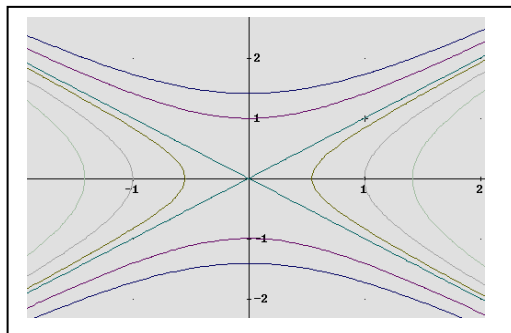
4. $t = 4$

5. $\frac{12}{17\sqrt{17}}$

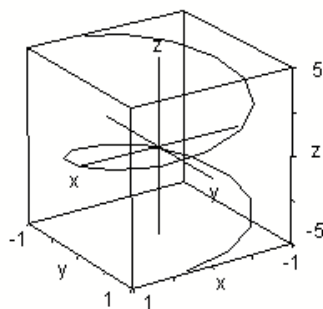
6. (a) D.N.E. (b) D. N. E. (c) 0 (d) 0

8. $x = 2 \sin \phi \cos \theta, y = 2 \sin \phi \sin \theta, z = 2 \cos \phi$
 $\pi/3 \leq \phi \leq 2\pi/3, 0 \leq \theta \leq 2\pi$

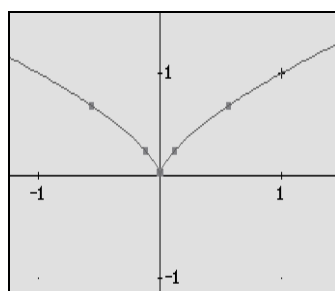
7.



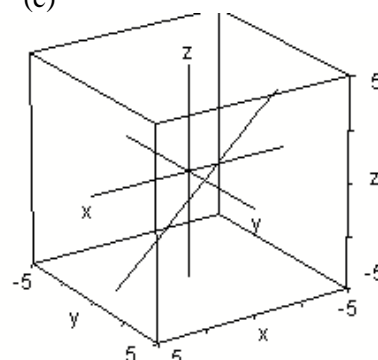
9. (a)



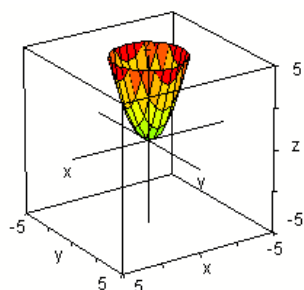
(b)



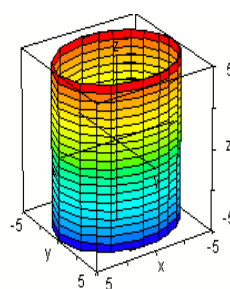
(c)



10. (a)



(c)



(b) upper semi-sphere of radius 2 centered at (0,0)

11. 0

12. 1

13. 4/3