Math 240 Review 2 Name:

You should go over other materials beside this review for the exam and exam2 covers chapter 10 and 11.1-11.3.

1. Consider the ellipse $\frac{x^2}{16}+\frac{y^2}{25}=1$.

(a) Find two different parametrizations of this ellipse.

(b) Find a point where the curvature is minimal. Show it both way, algebraically and graphically.

2. Find the equations for the following parametrized surfaces in rectangular coordinates, and describe them in words.

(a) $<t, \sqrt{1-t^2}\sin s, \sqrt{1-t^2}\cos s>$

(b) $<t, s, t^2+s^2>$

3. Find the parametric equations for the tangent line to the curve with the given parametric equations at the given point.

$x=\ln t, y=2\sqrt{t}, z=t^2 : (0,2,1)$

4. The position of a particle is given by $r(t)=<t^2, 5t, t^2-16t>$. When is the speed a minimum?

5. Find the curvature of the curve $y=x^4$ at the point (1,1).

6. Find the limit if exists.

(a) $\lim\limits_{(x,y)\to(0,0)} \frac{x^4+y^4}{(x^2+y^2)^2}$ (b) $\lim\limits_{(x,y)\to(1,1)} \frac{x-y^4}{x^3-y^4}$ (c) $\lim\limits_{(x,y)\to(0,0)} \frac{xy}{\sqrt{x^2+y^2}}$ (d) $\lim\limits_{(x,y)\to(0,0)} \frac{x^3+y^3}{x^2+y^2}$

7. Sketch the level curves of the function $z=f(x,y)=y^2-x^2$. Label your curves.

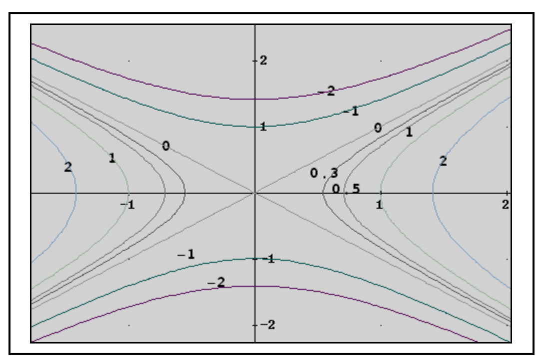
8. Find the parametric representation for the part of the sphere $x^2+y^2+z^2=4$ that lies between the planes $z=1$ and $z=-1$.

9. Sketch the following parametric curves.

(a) $x=\cos t, y=\sin t, z=t$

(b) $x=t^3, y=t^2$

(c) $x=1-t, y=2+t, z=2t$



10. Sketch the following parametric surfaces.

(a) $r(x,y)=<x, y, x^2+y^2>$

(b) $r(u,v)=<u, v, \sqrt{4-u^2-v^2}>$

(c) $r(u,v)=<5\cos u, 4\sin u, v>$

11. Estimate the value of $f_x(0,0)$ according to the contour map of the function $z=f(x,y)$ given above.

12. Find the value of $f_x(1,0)$ if $f(x,y)=x-4y^2$ using the limit definition and interpret the value as a slope.

13. Find the arc length of the curve given by $x=t, y=\frac{\sqrt{2}}{2}t^2, z=\frac{1}{3}t^3$ between the points (0,0,0) and $(1, \frac{\sqrt{2}}{2}, \frac{1}{3})$.

This is a brief outline of the main topics we had in class.

Sec 9.7 Spherical Coordinates
Rectangular Coordinates vs Cylindrical Coordinates vs Spherical Coordinates
Identify or sketch the surfaces or solids given by spherical coordinates

Chapter 10. Vector functions
Sec 10.1 Vector functions and Space Curves
Definition of vector functions (parametric equation) and Space Curves
Limit of vector functions
Graph of parametric curves
Find a vector function (parametric equation) of the kinds of space curves, helices, line segment, intersecting curves of two surfaces
Matching curves

Sec 10.2 Derivatives and Integrals of Vector Functions
Definition of Derivative of a vector functions
Find the derivatives using theorem 2
Unit Tangent Vector, Equation of tangent line to a curve
Derivative Rules
Integrals of vector functions and the fundamental theorem of Calculus for vector functions

Sec 10.3 Arc length and Curvature
Arc length formula
Arc length parametrization
Definition of Curvature, formula 9, formula 10(theorem), formula 11
Osculating Circles

Sec 10.4 Motion in Space: Velocity and Acceleration
Position vector, Velocity vector, Acceleration Vector
Newton’s Second Law of Motion

Sec 10.5 Parametric Surfaces
Definition of Parametric equation of Surfaces
Grid Curves
Parametric equations for several surfaces: Cylinder, Plane, Sphere, Cone, Paraboloid, Surface of Revolution
Matching Surfaces

Chapter 11. Partial Derivatives
Sec 11.1 Functions of Several Variables
Domain and Range of functions of several variables
Graph of functions of two variables
Level curves, contour map, contour lines(curves)
Matching of surfaces and contour map

Sec 11.2 Limits and Continuity
Definition of limits of $f(x, y)$ as (x, y) approaches (a,b)
Two-path test for nonexistence of limit
Several ways to find the limits: squeeze theorem, algebraic theorem, continuity

Sec 11.3 Partial Derivatives
Definition of Partial derivatives, Its Notations
Geometric Meanings: Understanding as slopes
Find the Partial Derivatives directly
Implicit partial Differentiation
Higher Derivatives and Clairaut’s theorem