This is only the review and besides this, you should redo all Hw, quiz problems with problems done in class. This exam is based upon sections 11.3-11.8, 12.1-12.3 and some theorems, facts, or formulas will be asked to prove, show or derive in the exam.

1. Answer the following based on the level curves.
(a) Suppose this a graph of a function $z=f(x, y)$ and sketch the gradient $\nabla f(1,1)$ carefully.


| u | $(1,0)$ | $(0,1)$ | $(1,1)$ | $(2,-1)$ | $(-1,1)$ | $(1,-1)$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $D_{u} f(1,1)$ |  |  |  |  |  |  |

Suppose the function is given by $f(x, y)=y^{2}-x^{2}$.
(b) What is the directional derivative of $f$ at $(1,1)$ in the direction of the gradient vector? Is your answer in (a) approximately close to your answer?
(c) Find the value of $D_{u} f(1,1)$ using the several choices of $u$ given above.

In which direction does f increase most rapidly? Decrease most rapidly? Or change nothing?
2. Suppose $u=<1,0>, v=<\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}>, D_{u} f(a, b)=3$ and $D_{v} f(a, b)=\sqrt{2}$.
(a) Find $\nabla f(a, b)$.
(b) What is the maximum possible value of $D_{w} f(a, b)$ for any w?
(c) Find a unit vector $w=<w_{1}, w_{2}>$ such that $D_{w} f(a, b)=0$.
3. Find the absolute maximum and minimum values of $f$ below on the rectangle shown below. Justify your answer.

(a) $f(x, y)=x y-2 x-2 y+4$.
(b) $f(x, y)=e^{-\left(x^{2}+y^{2}\right)}$.
4. Suppose that $f(x, y)=e^{x-y}$ and $f(\ln 2, \ln 2)=1$. Use the technique of linear approximation to estimate $f(\ln 2+0.1, \ln 2+0.04)$.
5. Consider the sphere $x^{2}+y^{2}+z^{2}=9$. Find the equation of the plane tangent to this sphere at $(2,2,1)$.
6. Evaluate the following double integral.
(a) $\iint_{R}(1-|x|)$ dxdy where $\mathrm{R}=[-1,1] \times[-2,3]$
(b) $\int_{-3}^{3} \int_{-2}^{2} \sqrt{4-y^{2}} d y d x$
(c) $\int_{0}^{6} \int_{x / 3}^{2} x \sqrt{y^{3}+1} d y d x$
(d) $\iint_{R} x y d A$ over $R=\left\{(x, y) \mid-\sqrt{1-x^{2}} \leq y \leq \sqrt{1-x^{2}}, x \geq 0\right\}$
7. The temperature at a point $(\mathrm{x}, \mathrm{y})$ is $\mathrm{T}(\mathrm{x}, \mathrm{y})$, measured in degrees Celsius. A bug crawls so that its position after t seconds is given by $x=\sqrt{1+t}, y=2+\frac{1}{3} t$, where x and y are measured in centimeters. The temperature function satisfies $T_{x}(2,3)=4$ and $T_{y}(2,3)=3$. How fast is the temperature rising on the bug's path after 3 second?
8. If $y z^{4}+x^{2} z^{3}=e^{x y z}$ and $z=f(x, y)$ is a function of x and y , find $\frac{\partial z}{\partial y}$ and $\frac{\partial z}{\partial x}$.
9. Suppose $f$ is differentiable function of $x$ and $y$, and $g(u, v)=f\left(e^{u}+\sin v, e^{u}+\cos v\right)$.

Use the table below to calculate $g_{v}(0,0)$.
10. Use Lagrange multipliers to find the maximum and minimum values of the function
$f(x, y)=x^{2} y ; x^{2}+y^{2}=1$
11. Consider the region R bounded by $y=x, y=-x+2, y=-\sqrt{1-(x-1)^{2}}$.

|  | f | g | $f_{x}$ | $f_{y}$ |
| :--- | :--- | :--- | :--- | :--- |
| $(0,0)$ | 3 | 6 | 4 | 8 |
| $(1,2)$ | 6 | 3 | 2 | 5 |

Set up the following integrals as one or more iterated integrals, but do not actually compute them.
(a) $\iint_{R}(x+y) d y d x$
(b) $\iint_{R}(x+y) d x d y$

This is a brief outline of the main topics we had in class.
Sec 11.4 Tangent Planes and linear approximations
Equation of tangent plane of a surface given by $r(u, v)$ and $z=f(x, y)$, Linear approximation, Differentials,
Applications of linear approximation
Sec 11.5 Chain Rules
Chain rules (case 1, case 2, general case), Implicit differentiation
Sec 11.6 Directional derivatives
Find the directional derivative using the limit definition, Find $D_{u} f(a, b)$ using the theorem 3, Gradient vector,
Gradient vector from contour map, Maximum and minimum directional derivatives (theorem 15 in page 795), Tangent line to level curve \& tangent plane to level surface

Sec 11.7 Maximum and Minimum Values
Critical points, Second derivative test, Local extremes and saddle point, Absolute extremes on closed and bounded domain

Sec 11.8 Lagrange multipliers
Lagrange multiplier of the function $z=f(x, y)$ or $w=f(x, y, z)$ with one constraint (one side condition)
Sec 12.1 Double integrals over a rectangle
Please remember that a rectangle $R=[a, b] \times[c, d]$, Definition of double integral of $z=f(x, y)$,
Understanding the double integral as volume, Midpoint rule, Average Value, Properties of double integrals
Sec 12.2 Iterated integrals
Fubini's theorem, Iterated integral, Different of order of integration, Using a double integral to get a volume, When can a double integral be written as the product of two single integrals?

Sec 12.3 Double integrals over general regions
Two types of regions, Setting up the upper and lower limits of double integrals according to the types of regions, Properties of double integral over general region

