Math 240 Review 4 Name:

Please go over all notes, worksheets, homework problems and quizzes related to sections 12.2 through 12.9 (skip 12.5).

1. Rewrite the integral $\int_0^{2\pi}\int_0^1 r^2 dr\ d\theta$ in rectangular coordinates.

2. Evaluate $\iint_D \cos(x^2+y^2)\text{dA}$ where $D=\{(x,y)\,|\,1\le x^2+y^2\le 4\}$.

3. Consider the region R bounded by $y=x,\ y=-x+2,\ y=-\sqrt{1-(x-1)^2}$.

Set up the following integrals as one or more iterated integrals, but do not actually compute them.

(a) $\iint_R (x+y)dy\ dx$

(b) $\iint_R (x+y)dx\ dy$

4. Set up and evaluate an integral giving the **surface area** of the surfaces given by

(a) $x=u+v,\ y=u-v,\ z=2u+3v,\ 0\le u\le 1, 0\le v\le 1.$

(b) $z=x^2+y$ above the triangle with vertices (0,0), (1,0) and (0,2).

5. Consider the triple integral $\int_0^1\int_{y^3}^{\sqrt{y}}\int_0^{xy} dz\ dx\ dy$ representing a solid. Let R be the projection of S onto the plane $z=0$.

(a) Draw the region R.

(b) Rewrite this integral as $\iiint_S dz\ dy\ dx$.

6. Consider the transformation T: $x=2u+v,\ y=u+2v$.

(a) Describe the image S under T of the unit square R=$[0,1]\times[0,1]$ in the uv-plane using a change of coordinates.

(b) Evaluate $\iint_S (3x+2y)dA$.

7. What is the volume of the following solid?

(a) $1\le\rho\le 9,\ 0\le\theta\le\pi/2,\ \pi/6\le\phi\le\pi/4$

(b) Bounded by the cylinder $x^2+y^2=4$ and the planes $z=0$ and $y+z=3$

(c) The solid tetrahedron with vertices (0,0,0),(0,0,1),(0,2,0) and (2,2,0)

8. Evaluate the triple integrals below.

(a) $\int_{-2}^{2}\int_0^{\sqrt{4-y^2}}\int_{-\sqrt{4-x^2-y^2}}^{\sqrt{4-x^2-y^2}} y^2\sqrt{x^2+y^2+z^2}\,dzdxdy$

(b) $\int_{-3}^{3}\int_0^{\sqrt{9-x^2}}\int_0^{9-x^2-y^2}\sqrt{x^2+y^2}\,dzdydx$

9. Evaluate the integral by making an appropriate change of variables.

$\iint_R e^{x+y}dA$, where R is given by $|x|+|y|\le 1$.

Answers

1. $\int_{-1}^{1}\int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}}\sqrt{x^2+y^2}\,dxdy$ 2. $\pi(\sin 4-\sin 1)$ 3(a) $\int_0^1\int_{-\sqrt{1-(x-1)^2}}^{x}(x+y)dydx+\int_1^2\int_{-\sqrt{1-(x-1)^2}}^{-x+2}(x+y)dydx$

(b) $\int_{-1}^{0}\int_{-\sqrt{1-y^2}+1}^{\sqrt{1-y^2}+1}(x+y)dxdy+\int_0^1\int_y^{2-y}(x+y)dxdy$ 4. (a) $\sqrt{30}$ (b) $\ln(\sqrt{2}+\sqrt{3})+\frac{\sqrt{2}}{3}$

5. (b) $\int_0^1 \int_{x^2}^{x^{1/3}} \int_0^{xy} dzdydx$ 6. (b) 45/2 7. (a) $\frac{182\pi}{3}(\sqrt{3}-\sqrt{2})$ (b)12π (c) 2/3

8.(a) $\frac{64\pi}{9}$ (b)$\frac{162\pi}{5}$ 9. $e-e^{-1}$

This is a brief outline of the main topics we had in class. Please make sure you know how to get those correctly.

12.2 Double integral over a rectangle
Understand $R=[a,b]\times[c,d]$
Fubini's theorem for double integral over a rectangle
Find the value of double integral by the volume of proper solid
Properties of double integrals

12.3 Double Integrals over General Regions
Understand type I, type II regions
Setting up double Integrals according to the type of regions
Find the volume of a solid by double integral
Change of order of integration
Properties of double integrals

12.4 Double Integrals in Polar Coordinates
Understand polar rectangles
Find the volume of the solid by polar coordinates
Evaluate double integral over a general region in polar coordinates

12.5 Skip

12.6 Surface Areas
Two formulas to get the surface areas: $\iint_{(u,v)} \| r_u \times r_v \| dudv$ or $\iint_{(x,y)} \sqrt{f_x^2 + f_y^2 + 1} dA$
Surface Area of surface of revolution

12.7 Triple Integrals
Fubini's theorem for triple integrals over a rectangular box
Triple integral over a general bounded region E
Choose the best order of integration
Find the volume by triple integrals

12.8 Triple Integrals in cylindrical and Spherical Coordinates
Review: Cylindrical and Spherical Coordinates
Evaluate the triple integrals in cylindrical coordinates
Evaluate the triple integrals in spherical coordinates

12.9 Change of Variables in Multiple Integrals
Transformations T
Understand the Jacobian J of the transformation T
Change of variables in double integrals, triple integrals