Math 240 Review 5 Name:

Please go over all notes, worksheets and homework problems in Chapter 13.

1. (a) Determine whether or not $F(x,y)=(y\cos x-\cos y)\mathrm{i}+(\sin x+x\sin y)\mathrm{j}$ is a conservative vector field. If it is, find a potential function f such that $\nabla f=F$.

(b) Evaluate the integral $\int_C F\cdot dr$ using part (a), where C is the line segment from $(0,\pi)$ to $(\pi,2\pi)$.

(c) Evaluate the integral $\int_C F\cdot dr$, where C is the ellipse $x^2+xy+y^2=1$.

2. Find the work done by the force field below on a particle that moves along the given curve.

(a) $F(x,y)=xy^2\mathrm{i}+y\mathrm{j}$ along the curve $y=x^2$ from (0,0) to (1,1).

(b) $\nabla(\frac{x^3}{3}+xy+\frac{y^3}{3})$ along a path given by $r(t)=\langle e^t\cos t,-e^{t-\frac{\pi}{2}}\sin t\rangle$ from the point (1,0) to (0,-1)

3. Let C be the closed path from (0,0) to (2,4) along $y=x^2$ and back again from (2,4) to (0,0) along $y=2x$. Evaluate $\int_C (x^3+2y)dx+(x-y^2)dy$ using Green's theorem.

4. Find the flux of $F(x,y,z)=x^2z\mathrm{i}-3xy^2\mathrm{j}+4y^3\mathrm{k}$, where S is the part of the elliptic paraboloid $z=x^2+y^2-9$ that lies below the square $0\le x\le 1,\quad 0\le y\le 1$ and has **downward orientation**.

5. Evaluate the following integrals.
You must sketch all necessary domains (or regions) of your integral.

(a) $\iint_S xzdS$, where S is the triangle with vertices (1,0,0), (0,1,0) and (0,0,1).

(b) $\iint_S \mathrm{F}\cdot d\vec{S}$, where $\mathrm{F}(x,y,z)=\langle x^2,xy,z\rangle$ and S is the part of the surface $z=x^2+y^2$ below the plane $z=1$, with upward orientation.

6. Use Stokes's theorem to evaluate $\oint_C F\cdot dr$, if $F=(xz)\mathrm{i}+(xy)\mathrm{j}+(3xz)\mathrm{k}$ and C is the boundary of the portion of the plane $2x+y+z=2$ in the first octant, traversed counterclockwise as viewed from above.

7. Use the Divergence theorem to calculate the surface integral $\iint_S F\cdot d\vec{S}$, where $F(x,y,z)=x^3\mathrm{i}+y^3\mathrm{j}+z^3\mathrm{k}$ and S is the surface of the solid bounded by the cylinder $x^2+y^2=1$ and the planes $z=0$ and $z=2$.

8. Use the theorems we had in class to find the following. Provide the name of theorem applied.

(a) $\iint_S F\cdot d\vec{S}$, where $F(x,y,z)=(x^2+y^2)\mathrm{i}+(y^2+z^2)\mathrm{j}+(z^2+x^2)\mathrm{k}$ through the cube $[0,1]\times[0,1]\times[0,1]$

(b) $\iint_S \mathrm{curl}F\cdot d\vec{S}$, $F(x,y,z)=-y\mathrm{i}+x\mathrm{j}+(xy+\cos z)\mathrm{k}$ and S is the disk $x^2+y^2\le 9$, oriented upward.

(c) $\int_C \mathrm{grad}f\cdot dr$, where $f(x,y)=3x+4y$ and C is a line segment of length 10 in the plane starting at (2,1).

9. Show that the line integral $\oint_C F\cdot dr$ of vector field $F(x,y)=y\cos x\mathrm{i}+(\sin x+y)\mathrm{j}$ is independent of path.

10. Prove: $div(\nabla f\times\nabla g)=0$ for two real valued functions f and g.

Answer key
1. (a) $f(x,y)=y\sin x-x\cos y+c$ (b) $-\pi$ (c) 0
2. (a) 2/3 (b) -2/3
3. -4/3
4. -23/4
5. (a) $\sqrt{3}/24$ (b) $\pi/2$
6. -1
7. 11π
8. (a) 3 (b) 18π (c) it varies
9. 10. Leave it to you!

13. Vector Calculus

13.1 Vector Fields

Definition of vector fields
Graph of vector fields
Gradient fields, conservative vector fields F, potential function for F

13.2 Line Integrals

Definition of the line integral of f along a curve C: $\int_C f(x,y)\,ds$
Line integrals of f along C with respect to x and y
Line integral of vector fields: $\int_C F\cdot dr$
Find the amount of work done by a force field F along a curve C

13.3 The Fundamental Theorem of Line Integrals

Statement of FTOL
Evaluate the line integrals using FTOL
Path independent line integrals
Find the potential function for a conservative vector field

13.4 Green's Theorem

Statement of Green's theorem
Evaluate the line integrals using Green's theorem
Find the area using Green's theorem

13.5 Curl and Divergence

Definition of curl and divergence of vector fields
Some properties

13.6 Surface Integrals

Definition of surface integrals for a function f and a vector field
Flux of F over a surface S

13.7 Stokes' theorem

Statement of Stokes' theorem
Evaluate the line integral on C or surface integral using Stokes' theorem

13.8 Divergence Theorem

Statement of Divergence theorem
Evaluate the surface integral of F over S using divergence theorem