Math 286(5.1 & 5.2) Hw 10 Name & Box number:

1. Determine the radius of convergence and interval of convergence of the given power series. Check the end points as well!

(a) $\sum_{n=0}^{\infty} 2^n x^n$ (b) $\sum_{n=1}^{\infty} \frac{(2x+1)^n}{n^3}$ (c) $\sum_{n=1}^{\infty} \frac{(-1)^n n^2 (x+1)^n}{3^n}$

2. Determine the Taylor series about the point $x_0$ for the given function. Also, determine the radius of convergence of the series.

(a) $\sin x$ at $x_0 = 0$ (b) $\frac{1}{1+x}$ at $x_0 = 0$ (c) $\frac{1}{1-x}$ at $x_0 = 2$

3. Combine the three series in the following questions into a single summation in $x^n$ by shifting the index of summation whenever necessary and performing the indicated additions and subtractions. (Write out some of the terms individually when they cannot be combined into the summation.)

(a) $\sum_{n=0}^{\infty} nC_n x^n - 2\sum_{n=0}^{\infty} (n+1)C_n x^{n+2} + x\sum_{n=0}^{\infty} n2^n C_n x^{n+1} = 0$

(b) $\sum_{n=1}^{\infty} n^2 x^{n-1} - 5x\sum_{n=0}^{\infty} (n-3)C_n x^{n-2} + x^2\sum_{n=0}^{\infty} \frac{n-1}{n+1} x^{n-4} = 0$

(c) $\sum_{n=0}^{\infty} (-1)^n (n+2)C_n x^{n+1} + 2\sum_{n=1}^{\infty} n^3 x^{n+2} - 3x^2\sum_{n=0}^{\infty} nx^n = 0$

4. Find the power series solutions of the given DE about $x_0 = 0$.

(i) First find the recurrence relation.

(ii) Find the first four terms in each of two solutions $y_1$ and $y_2$.

(iii) Find the Wronskian of $y_1$ and $y_2$ to show that they are fundamental solutions.

(iv) If possible, find the general term in each solution.

(a) $y'' - y = 0$ (b) $y'' + k^2 x^2 y = 0$ and k is a constant.

(c) $(4 - x^2)y'' + 2y = 0$ (d) $2y'' + xy' + 3y = 0$