Math 286(sec 2.2, 2.3) Hw 2 Name & Box Number:

Show all your work. Points will not be given only for answers.

For problems 1 & 2,

(a) Using the link available at the class website, sketch the direction field for the given differential equation.

(b) Based on an inspection of the direction field, describe how solutions behave for large t.

(c) Find the general solution of the given differential equation, and use it to determine how solutions behaves as $t \to \infty$.

1. $ty' + 2y = \sin t$

2. $2y' + y = 3t^2$

3. Find the solution of the given initial value problem IVP.

(a) $y' + 2y = te^{-2t}, \quad y(1) = 0$

(b) $y' + (2/t)y = (\cos t)/t^2, \quad y(\pi) = 0$

(c) $t^3 y' + 4t^2 y = e^{-t}, \quad y(-1) = 0$

(d) $ty' + (t+1)y = t, \quad y(\ln 2) = 1$

4. Find the value of y_0 for which the solution of the IVP: $y' - y = 1 + 3\sin t, \quad y(0) = y_0$ remains finite as $t \to \infty$.

5. A tank initially contains 120 L of pure water. A mixture containing a concentration of r gal/L of salt enters the tank at a rate of 2L/min, and the well stirred mixture leaves the tank at the same rate. Find an expression in terms of r for the amount of salt in the tank at any time t. Also find the limiting amount of salt in the tank as $t \to \infty$.

6. A tank contains 100 gal of water and 50 oz of salt. Water containing a salt concentration of $\frac{1}{4}(1 + \frac{1}{2}\sin t)$ oz/gal flows into the tank at a rate of 2 gal/min, and the mixture in the tank flows out at the same rate. Find the amount of salt in the tank at any time and plot the solution for a period long enough so that you see the ultimate behavior of the graph.

7. A 120 gallon tank initially contains 90 lbs of salt dissolved in 90 gal of water. Brine containing 2 lb/gal of salt flows into the tank at the rate of 4 gal/min, and the well stirred mixture flows out of the tank at the rate of 3 gal/min. How much salt does the tank contain when it is full.

8. Determine (without solving the problems) an interval in which the solution of the given IVP is certain to exist.

(a) $t(t-4)y' + y = 0, \quad y(2) = 1$

(b) $(\ln t)y' + y = \cot t, \quad y(4) = 3$