

1. Solve the second order linear DE or IVP given below.

(a) $y'' + 3y' + 2y = 0$

(b) $4y'' - y = 0, \quad y(-2) = 1, \quad y'(-2) = -1$

2. Find the solution of the IVP $2y'' - 3y' + y = 0, \quad y(0) = 2, \quad y'(0) = \frac{1}{2}$. Then determine the maximum value of the solution and also find the point where the solution is zero.

3. Solve the IVP $4y'' - y = 0, \quad y(0) = 2, \quad y'(0) = \beta$. Then find β so that the solution approaches zero as $t \rightarrow \infty$.

4. Determine the longest interval in which the given initial value problem is certain to have a unique solution. Use the theorem 3.2.1, Existence and Uniqueness Theorem.

(a) $(t-1)y'' - 3ty' + 4y = \sin t, \quad y(-2) = 2, \quad y'(-2) = 1$

(b) $y'' + (\cos t)y' + 3(\ln |t|)y = 0, \quad y(2) = 2, \quad y'(2) = 1$

5. Determine if the following second-order differential equations are (1) linear or non-linear, (2) homogeneous or nonhomogeneous, and (3) with constant or variable coefficients.

(a) $y'' + 3yy' = 6x^2$ (b) $y'' - 3y = e^{2x}$ (c) $x^2y'' + xy' + y = 0$ (d) $y'' + xy' - 3y = \sin 2x$

6. Verify that the functions y_1 and y_2 are solutions of the given DE. Do they constitute a fundamental set of solutions?

$y'' - 2y' + y = 0, \quad y_1(t) = e^t, \quad y_2(t) = te^t$