

1. Solve the DE or IVP given below. For IVP, sketch the graph of the solution and describe its behavior for increasing t .

(a) $y'' - 2y' + 6y = 0$

(b) $4y'' + 9y = 0$

(c) $y'' + y' + 1.25y = 0$, $y(0) = 3$, $y'(0) = 1$

(d) $y'' - 2y' + 5y = 0$, $y(\frac{\pi}{2}) = 0$, $y'(\frac{\pi}{2}) = 2$

(e) $y'' - 6y' + 9y = 0$

(f) $9y'' - 12y' + 4y = 0$, $y(0) = 2$, $y'(0) = -1$

(g) $t^2 y'' + 3ty' - 3y = 0$, $t > 0$

(h) $t^2 y'' + 7ty' + 10y = 0$, $t > 0$

2. Find the Wronskian of the given pair of the functions.

(a) x, xe^{2x}

(b) $e^t \sin t, e^t \cos t$

(c) $\cos^2 \theta, 1 + \cos 2\theta$

3. Without solving the equations find the Wronskian of two solutions of the given DE below.

(a) $t^2 y'' - t(t+2)y' + (t+2)y = 0$

(b) $x^2 y'' + xy' + (x^2 - \nu^2)y = 0$ for a constant ν (it is called the Bessel equation)

(c) $(1-x^2)y'' - 2xy' + \alpha(\alpha+1)y = 0$ for a constant α (it is called the Legendre equation)

4. Use the method of reduction of order to find the second solution of the DE below.

(a) $t^2 y'' - t(t+2)y' + (t+2)y = 0$, $t > 0$; $y_1(t) = t$

(b) $xy'' - y' + 4x^3 y = 0$, $x > 0$; $y_1(t) = \cos x^2$

5. Consider the IVP: $4y'' + 12y' + 9y = 0$, $y(0) = 1$, $y'(0) = b$. Find the value of b that separates solutions that always remain positive from those that eventually become negative.