

1. Find the general solution of the given differential equation or solution to IVP.

(a) $y'' + 2y' = 3 + 4\sin 2t$

(b) $y'' + 9y = t^2 e^{3t} + 6$

(c) $y'' + 4y = t^2 + 3e^t$, $y(0) = 0$, $y'(0) = 2$

(d) $y'' - 2y' - 3y = 3te^{2t}$, $y(0) = 1$, $y'(0) = 0$

(e) $y'' + 4y = 3\csc 2t$, $0 < t < \pi/2$

(f) $y'' - 5y' + 6y = g(t)$, $g(t)$ is an arbitrary continuous function

2. Verify that the given functions y_1 and y_2 satisfy the corresponding homogeneous equation; then find a particular solution of the given nonhomogeneous equation.

(a) $t^2 y'' - 2y = 3t^2 - 1$, $t > 0$; $y_1 = t^2$, $y_2 = t^{-1}$

(b) $x^2 y'' - 3xy' + 4y = x^2 \ln x$, $x > 0$; $y_1 = x^2$, $y_2 = x^2 \ln x$

(c) $(1-x)y'' + xy' - y = g(x)$, $0 < x < 1$; $y_1 = e^x$, $y_2 = x$, $g(x)$ is an arbitrary continuous function.

3. Determine a suitable form for $Y(t)$ if the method of undetermined coefficients is to be used and use a computer algebra system to find a particular solution of the given equation.

$$y'' + 4y = t^2 \sin 2t + (6t + 7)\cos 2t$$

4. Consider the vibrating system described by the initial value problem

$$u'' + u = 3\cos \omega t, \quad u(0) = 1, \quad u'(0) = 1$$

(a) Find the solution for $\omega \neq 1$.

(b) Plot the solution $u(t)$ for $\omega = 0.7, 0.8, 0.9$.

(c) What happens to the solutions if we consider different initial conditions $u(0) = 0$, $u'(0) = 0$ for the same differential equation?

Use a computer software and attach the prints for (b) and (c).