

1. Determine the general solution of the second-order Euler equation below.

$$2x^2 y'' + 6xy' + 2y = 4x^2$$

2. A mass of 5 kg stretches a spring 10cm. The mass is acted on by an external force of $10\sin(t/2)$ N (newtons) and moves in a medium that imparts a viscous force of 2 N when the speed of the mass is 4cm/s. If the mass is set in motion from its equilibrium position with an initial velocity of 3cm/s, then the equation of motion satisfies

$$5u'' + 50u' + 490u = 10\sin(t/2), \quad u(0) = 0 \text{ m}, \quad u'(0) = 0.03 \text{ m/s}.$$

(a) Find the solution of the IVP.

(b) Identify the transient and steady state parts of the solution.

(c) Plot the graph of the steady state solution using any computer software.

3. Find a general solution to the following higher order equations.

(a) $y^{(4)} - y^{(3)} - 3y'' + 5y' - 2y = 0$

(b) $y''' - 4y' = t$

(c) $y''' + y'' + 3y' - 5y = 0$