Math 286

Review for Exam 1

This exam covers from section 2.1 thru 2.7. It is only a review and you also should go over other materials carefully for the test.

1. Find the general solutions of the following. Tell the type of the differential equation.

(a) $x^3+3y-xy'=0$ (b) $xy^2+3y^2-x^2y'=0$ (c) $xy+y^2-x^2y'=0$

(d) $2xy^3+e^x+(3x^2y^2+\sin y)y'=0$ (e) $3y+x^4y'=2xy$

(f) $2xy^2+x^2y'=y^2$ (g) $2x^2y+x^3y'=1$ (h) $2xy+x^2y'=y^2$

(i) $xy'+2y=6x^2\sqrt{y}$ (j) $y'=1+x^2+y^2+x^2y^2$ (k) $x^2y'=xy+3y^2$

(l) $6xy^3+2y^4+(9x^2y^2+8xy^3)y'=0$ (m) $xy'=6y+12x^4y^{2/3}$

2. A cup of soup is initially 150 degree. Suppose that it cools to 140 degree in 1 minute in a room with an ambient temperature of 70 degree. Assume that Newton's law of cooling applies: The rate of cooling is proportional to the difference between the current temperature and the ambient temperature.

(a) Write an initial value problem that models the temperature of the soup.

(b) How long does it take the soup to cool to a temperature of 100 degree?

3. Consider the initial value problem $dy/dt=y^2-2y+1, y(0)=2$.

(a) Use Euler's method with $h=\Delta t=0.5,$ graph an approximation solution over the interval $0\le t\le 2.$

(b) What happens when you try to repeat part (a) with $h=\Delta t=0.05?$

(c) Solve the above initial value problem, and use the result to explain your observations in part (a) and (b)?

4. Sketch the phase lines for the given differential equation. Identify the equilibrium points as stable, or unstable.

(a) $dy/dt=3y(y-1)$ (b) $dy/dt=y\ln|y|$

5. Use Existence and Uniqueness theorem for linear first order differential equation or nonlinear first order differential equation to determine the following differential equation has a unique solution and find the valid interval of the solution.

(a) $y'=y^2, y(0)=1$

(b) $(t-3)y'+(\ln t)y=2t, y(1)=2$

6. Consider the differential equation $dy/dt=-2ty^2$.

(a) Find its general solution. (b) Find all values of y_0 such that the solution to the IVP $dy/dt=-2ty^2, y(-1)=y_0$ does not blow up (or down) in finite time. i.e. find all y_0 such that the solution is defined for all value of t.

Some selected answer

1. (a) Linear: $y = x^3(C + \ln x)$ (b) Separable $y = x/(3 - Cx - x\ln x)$
(c) Homogeneous $y = x/(C - \ln x)$ (d) Exact: $x^2y^3 + e^x - \cos y = C$
(e) Separable: $y = Ce^{(x^{-3} - x^{-2})}$ (f) Separable: $y = x/(1 + Cx + 2x\ln x)$
(g) Linear: $y = x^{-2}(C + \ln x)$ (h) Homogeneous: $y = 3Cx/(C - x^3)$
(i) Bernoulli: $y = (x^2 + Cx^{-1})^2$ (j) Separable: $y = \tan(C + x + 1/3x^3)$
(k) Homogeneous: $y = x/(C - 3\ln x)$ (l) Exact: $3x^2y^3 + 2xy^4 = C$
(m) Bernoulli: $y = (2x^4 + Cx^2)^3$

2. (a) $dT/dt = k(T - 70), T(0) = 150$ (b) 7.3 minutes

3. (c) $y(t) = (t-2)/(t-1);$ the solution blows up when t =1 in finite time

4. (a) $y = 0$ stable; $y = 1$ unstable (b) The equation is not defined at y=0, but near y=0 the solution act as if they are stable at y=0; y=-1, y=1 unstable

5. (a) $-\infty < t < 1$ (b) $0 < t < 3$

6. (a) $y(t) = 1/(t^2 + k)$ and equilibrium solution y = 0. (b) $0 \le y_0 < 1$