Review for Exam 2
Math 286

This exam will cover sections 3.1 thru 3.10 and Chapter 4(brief).
You should go over all the quiz, worksheet, note and Homework problems with this review for the test.

1. Solve the following second order DE or IVP.
(a) $y''+11y'+24y=0; y(0)=1, y'(0)=4$
(b) $y''-\frac{7}{x}y'+\frac{16}{x^2}y=0; y(1)=2, y'(1)=4$ with the two solutions $y_1=x^4, y_2=x^4\ln x$
(c) $y''+9y=0; y(\frac{\pi}{3})=1, y'(\frac{\pi}{3})=-2$ (d) $y''-14y'+49y=0$ (e) $x^2y''-5xy'+9y=0$
(f) $x^2y''+xy'-16y=0$ (g) $x^2y''-5xy'+8y=2\ln(x)$

2. Find the general solution of $y''-(3/x)y'+(4/x^2)y=0$ for x > 0, given that one solution is $y_1=x^2$.

3. Consider the vibrating system described by the IVP: $u''+u=3\cos\varpi t, u(0)=1, u'(0)=1.$
Find the solution for $\varpi\neq1$.

4. The position of a certain undamped spring-mass system satisfies $u''+2u=0, u(0)=0, u'(0)=2$.
Find the solution.

5. Write down the following below.
(a) State the Principle of Superposition and prove it.
(b) Fundamental set of solutions to $y''+p(t)y'+q(t)y=0$.
(c) State the Existence and Uniqueness theorem for the second order linear differential equation.
(d) Prove: If y_1 and y_2 are particular solution to the non-homogeneous equation
$y''+p(t)y'+q(t)y=g_1(t)$ and
$y''+p(t)y'+q(t)y=g_2(t)$, respectively, then y_1+y_2 is a particular solution to
$y''+p(t)y'+q(t)y=g_1(t)+g_2(t)$.
(e) Abel's Theorem and its proof

6. Find a general solution of the differential equation below.
(a) $y''-y'-2y=5e^{4x}+6x$
(b) $y''+8y'+16y=x\cos(2x)$

7. Write down the guess for the particular solution to the given differential equation
Do not find the coefficients.
(a) $y''-4y'+4y=2t^2+4te^{2t}+t\sin t$
(b) $y''+8y'+16y=e^{-4t}+(t^2+5)e^{-4t}$

8. Find the general solution of the following higher order differential equations.
(a) $y'''+y''+3y'-5y=0$
(b) $y'''+3y''-4y=e^{-2t}$

9. Find a general solution to the DE using the method of variation of parameters.
$y''(\theta)+16y(\theta)=\sec(4\theta)$

Answer key to Review2-Math 286

1. (a) $y=\frac{12}{5}e^{-3x}-\frac{7}{5}e^{-8x}$ (b) $y=2x^4-4x^4\ln x$ (c) $y=-\cos(3x)+\frac{2}{3}\sin(3x)$ (d) $y=c_1e^{7t}+c_2te^{7t}$

(e) $y=c_1x^3+c_2x^3\ln x$ (f) $y=c_1x^4+c_2x^{-4}$ (g) $y=c_1x^2+c_2x^4+\frac{1}{4}\ln(x)+\frac{3}{16}$

2. $y=c_1x^2+c_2x^2\ln(x)$

3. $u=[(w^2+2)\cos t-3\cos\varpi t]/(\varpi^2-1)+\sin t$

4. $u=\sqrt{2}\sin\sqrt{2}t$

5. Please find them from your textbook.

6. (a) $y=c_1e^{-x}+c_2e^{2x}+\frac{1}{2}e^{4x}-3x+\frac{3}{2}$

(b) $y=c_1e^{-4x}+c_2xe^{-4x}+\frac{3}{100}x\cos(2x)+\frac{1}{25}x\sin(2x)-\frac{1}{250}\cos(2x)-\frac{11}{500}\sin(2x)$

7. (a) $y=At^2+Bt+C+t^2(Dt+E)e^{2t}+(Ft+G)\sin t+(Ht+L)\cos t$

(b) $y=t^2(At^2+Bt+C)e^{-4t}$

8. (a) $y=c_1e^t+c_2e^{-t}\cos 2t+c_3e^{-t}\sin 2t$ (b) $y=c_1e^t+c_2e^{-2t}+c_3te^{-2t}-\frac{1}{6}t^2e^{-2t}$

9. $y=c_1\cos(4\theta)+c_2\sin(4\theta)+\frac{\cos(4\theta)}{16}\ln|\cos(4\theta)|+\frac{\theta}{4}\sin(4\theta)$