Review for exam 3
Math 286

This exam will include chapter 5 and sections 8.1 – 8.9. You should go over all the quiz, worksheet, notebook and homework problems with this review for the test.

1. Find the first six nonzero terms in the series solution about zero.
If possible, use the $\sum$ notation.
(a) $y''+x^2y=0$ (b) $y''+x^2y'+4y=1-x^2$ (c) $y''+xy'+y=0$ (d) $(1-x)y''+xy'-y=0$

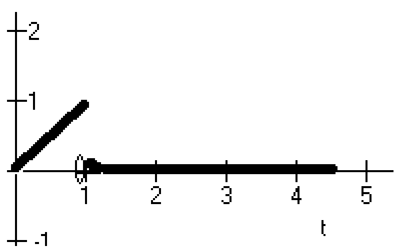
2. Consider the differential equation $x^2y''+x(\frac{1}{2}+2x)y'+(x-\frac{1}{2})y=0$.
(a) Show that 0 is a regular singular point.
(b) Find the indicial equation and get the value of r's.
(b) Find the Frobenius series solution.

3. Answer the same question in #2 for the following differential equation.
(a) $x^2y''+5xy'+(x+4)y=0$
(b) $x^2y''+x^2y'-2y=0$

4. Consider the Bessel equation of order zero $x^2y''+xy'+x^2y=0$ and find one solution.

5. Use the integration to find the Laplace transform of $e^t\sin 2t$.

6. Find the Laplace transform of the following function. Don't forget the domain.
(a) $f(t)=\sin 3t+\sinh 3t$ (b) $g(t)=-4t^3+3t^5$ (c) $h(t)=1-u_3(t)$
(d) (e) $f(t)=(t-3)u_2(t)$



7. Find the inverse Laplace transform of the following.
(a) $F(s)=\dfrac{3}{s^4}$ (b) $F(s)=\dfrac{9+s}{4-s^2}$ (c) $F(s)=2s^{-1}e^{-3s}$ (d) $F(s)=\dfrac{s^3}{(s-4)^4}$
(e) $H(s)=\dfrac{3s}{(s^2+1)(s^2+4)}$ (f) $F(s)=\dfrac{2(s-1)e^{-2s}}{s^2-2s+2}$

8. Solve the following IVP using the Laplace transforms.
(a) $y''+4y=0, y(0)=5, y'(0)=0$
(b) $y''-y'-2y=0; y(0)=0, y'(0)=2$
(c) $y''+y=\cos 3t; y(0)=1, y'(0)=0$
(d) $y^{(4)}-y=u_1(t)-u_2(t); \quad y(0)=y'(0)=y''(0)=y'''(0)=1$

Answer key to Review 3-Math 286

1.(a) $y(x)=a_0(1-\frac{1}{12}x^4+\frac{1}{672}x^8+...)+a_1(x-\frac{1}{20}x^5+\frac{1}{1440}x^9+...)$

(b) $y=a_0+a_1x+(\frac{1}{2}-2a_0)x^2-\frac{2}{3}a_1x^3+(-\frac{1}{4}+\frac{2}{3}a_0-\frac{1}{12}a_1)x^4+...$

(c) $y=a_0\sum_0^\infty(-1)^n x^{2n}/(n!2^n)+a_1\sum_0^\infty(-1)^n x^{2n+1}/(2n+1)!!$, $(2n+1)!!=1\cdot3\cdot5\cdot7...(2n+1)$

(d) $y_1=1+\frac{x^2}{2}+\frac{x^3}{6}+\frac{x^4}{24}+...+\frac{x^n}{n!}+...,y_2=x$

2. (a) leave it to you!

(b) -1/2, 1

(c) $y_1=a_0(x-\frac{6}{5}x^2+\frac{6}{7}x^3-\frac{4}{9}x^4+...),\ \ y_2=a_1x^{-1/2}$

3. (a) r = -2, double root $y_1=a_0\sum_0^\infty(-1)^n\frac{1}{(n!)^2}x^{n-2}$

(b) $r=-1,2$, $y_1=a_0x^2(1-\frac{1}{2}x+\frac{3}{20}x^2-\frac{1}{30}x^3+\frac{1}{168}x^4+...)$, $y_2=a_1(\frac{1}{x}-\frac{1}{2})$

4. $y_1=a_0(1-\frac{x^2}{4}+\frac{x^4}{64}-\frac{x^6}{2304}+...)=a_0(1+\sum_1^\infty\frac{(-1)^m x^{2m}}{2^{2m}(m!)^2})$

5. $\frac{2}{(s-1)^2+4}, s>1$

6. (a) $\frac{3}{s^2+9}+\frac{3}{s^2-9}, s>3$ (b) $-4\frac{3!}{s^4}+3\frac{5!}{s^6}, s>0$ (c) $\frac{1}{s}-\frac{e^{-3t}}{s}, s>0$

(d) $\frac{1-e^{-s}-se^{-s}}{s^2}, s>0$ (e) $\frac{e^{-2s}-se^{-2s}}{s^2},\ s>0$

7. (a) $\frac{1}{2}t^3$ (b) $\frac{-9}{2}\sinh 2t-\cosh 2t$ (c) $2u_3(t)$ (d) $e^{4t}(1+12t+24t^2+\frac{32}{3}t^3)$

(e) $\cos t-\cos 2t$ (f) $2u_2(t)e^{t-2}\cos(t-2)$

8. (a) $5\cos 2t$ (b) $\frac{2}{3}(e^{2t}-e^{-t})$ (c) $\frac{1}{8}(9\cos t-\cos 3t)$

(d) $-[u_1(t)-u_2(t)]+\frac{1}{4}[e^{-(t-1)}+e^{t-1}+2\cos(t-1)]u_1(t)-\frac{1}{4}[e^{-(t-2)}+e^{t-2}+2\cos(t-2)]u_2(t)+e^t$