Math 408(Chapter 1) Homework 1 Name & Number:

Due: Tuesday, Jan 17

1. Prove the following.

(a) $|(2\overline{z}+5)(\sqrt{2}-i)| = \sqrt{3}\,|2z+5|$

(b) $\overline{z^4} = (\overline{z})^4$

(c) $\overline{\left(\dfrac{z_1}{z_2 z_3}\right)} = \dfrac{\overline{z_1}}{\overline{z_2}\,\overline{z_3}}$

2. Solve the equation $z^2 + z + 1 = 0$ for $z = (x, y)$ by writing $(x, y)(x, y) + (x, y) + (1, 0) = (0, 0)$.

3. Show that when $|z_3| \neq |z_4|$, $\left|\dfrac{z_1 + z_2}{z_3 + z_4}\right| \leq \dfrac{|z_1| + |z_2|}{||z_3| - |z_4||}$.

4. In each case, sketch the set of points determined by the given condition.

(a) $|2z - i| = 4$

(b) $\text{Re}(\overline{z} - i) = 2$

(c) $|z + i| \geq 1$

5. Show that if z lies on the circle $|z| = 2$, $\left|\dfrac{1}{z^4 - 4z^2 + 3}\right| \leq \dfrac{1}{3}$

6. Use the Euler's formula to do the following. Write your final answer in terms of rectangular coordinates.

(a) $i(1-\sqrt{3}i)(\sqrt{3}+i) = 2(1+\sqrt{3}i)$

(b) $(-1+i)^7 = -8(1+i)$

(c) $\dfrac{5i}{2+i}$

7. Show that when $|z| < 1$, $|\text{Im}(1 - \overline{z} + z^2)| < 3$

8. Find all values for the following roots.

(a) $i^{1/2}$

(b) $(-8-8\sqrt{3}i)^{1/4}$

9. Show that if $\text{Re}\, z_1 > 0,\ \text{Re}\, z_2 > 0$, then $Arg(z_1 z_2) = Arg(z_1) + Arg(z_2)$.