Math 408(94, 96) Homework 10 Name & Number:

1. Use Rouche's theorem to Determine the number of zeros, counting multiplicities, of the polynomial below inside the circle $|z|=2$.

(a) $z^4+3z^3+6=0$
(b) $z^5+3z^3+z^2+1=0$

2. Determine the number of zeros, counting multiplicities, of the function $2z^5-6z^2+z+1=0$ in the annulus $1\le|z|<2$.

3. Find a linear transformation that maps the strip $x>0, \quad 0<y<2$ onto the strip $-1<u<1,\ v>0$.

4. (a) Find and sketch the region onto which the half plane $y>0$ is mapped by the transformation $w=(1+i)z$.
(b) Same as in (a) with $y>1$ by the transformation $w=(1-i)z$.

5. If $P(z)=a_0+a_1z+...+a_nz^n$, evaluate $\frac{1}{2\pi i}\int_{|z|=R}\frac{zP'(z)}{P(z)}\,dz$ for large values of R.
Hint: Use the fundamental theorem of algebra for this problem!

6. Use the Rouche's theorem to prove the fundamental theorem of algebra.
(example 2 in page 292)