Math 408(97-105) Homework 11 Name & Number:

1. (a) Show that the transformation $w = i\frac{1-z}{1+z}$ maps the disk $|z| \le 1$ onto the plane $\operatorname{Im} w \ge 0$.

(b) Use the result of (a) to verify that the LFT $w = \frac{z-2}{z}$ maps the disk $|z-1| \le 1$ onto the left half plane $\operatorname{Re} w \le 0$. Hint: $w = \frac{z-2}{z}$ can be written $Z = z-1$, $W = i\frac{1-Z}{1+Z}$, $w = iW$.

2. Find the LFT that maps
(a) $-i, 0, i$ to $-1, i, 1$.
(b) distinct z_1, z_2 and z_3 onto the points $0, 1, \infty$.

3. A fixed point of a transformation $w = f(z)$ is a point z_0 such that $f(z_0) = z_0$. Show that every LFT, with the exception of the identity transformation $f(z) = z$ has at most two fixed points in the extended plane.

4. Find the fixed points of the transformation
(a) $w = \frac{z-1}{z+1}$ (b) $w = \frac{6z-9}{z}$

5. Find the image of the region below under the transformation $w = \frac{1}{z}$.
(a) $x > 1,\ y > 0$
(b) $0 < y < 1/(2c)$, $c > 0$

6. Show that under the transformation $w = \sin z$, a line $x = c_1$ ($\frac{\pi}{2} < c_1 < \pi$) is mapped onto the right-hand branch of the hyperbola. Note that the mapping is 1-1 and that the upper and lower halves of the line are mapped onto the lower and upper haves, respectively, of the branch.

7. Show that the function $w = z^2$ maps the hyperbolas $x^2 - y^2 = c$ and $xy = k$ onto straight lines.