Math 408(13-22) Homework 2 Name & Number:

Due: Tues, Jan 24

1. Show that $w = f(z) = |z|^2 = x^2 + y^2$ is not differentiable at any point $z_0 \neq 0$.

2. Show that $w = f(z) = \begin{cases} \frac{(\overline{z})^2}{z} & \text{if } z \neq 0 \\ 0 & \text{if } z = 0 \end{cases}$ satisfies the CR equation at 0 but it is not differentiable at zero.

3. Determine whether $f'(z)$ exits and find its value when

(a) $f(z) = x^2 + iy^2$

(b) $f(z) = z \operatorname{Im} z$

4. Consider the mapping $w = f(z) = z^2$. Find the region in the first quadrant of the z-plane that mapped onto the unit square S: $0 \leq u \leq 1,\ 0 \leq v \leq 1$ of the w-plane.

5. Consider the mapping $w = f(z) = z + \frac{1}{z}$. Find the image of the top half of the unit disk.

6. Use the properties of derivatives to find $f'(z)$ when

(a) $f(z) = (1 - 4z^2)^3$

(b) $f(z) = \frac{(1+z^2)^4}{z^2}$

7. Compute the following limits.

(a) $\lim_{z \to i} \frac{z^2 + 1}{iz - 1}$

(b) $\lim_{z \to z_0} \frac{P(z)}{Q(z)}$, where P(z) and Q(z) are polynomial functions of z and $Q(z_0) \neq 0$.