Math 408(23-27) Homework 3 Name & Number:
Due: Jan 31, 2017

1. (a) Suppose that $w = f(z) = u(x,y) + iv(x,y)$ satisfies the polar CR-equation at z_0. Show that it satisfies the CR-equation at z_0. Therefore, if u_r, u_θ, v_r and v_θ are all continuous at z_0, then so are u_x, u_y, v_x and v_y which means $w = f(z)$ is differentiable at z_0.

(b) Show that $f'(z_0) = e^{-i\theta}(u_r + iv_r)$ in polar coordinates.

2. Suppose that a function $f(z) = u(x,y) + iv(x,y)$ and its conjugates $\overline{f(z)} = u(x,y) - iv(x,y)$ are both analytic in a given domain D. Show that $f(z)$ must be a constant throughout D.

3. Apply the theorem in the class to verify that each of the following is entire.
(a) $f(z) = e^{-y}\sin x - ie^{-y}\cos x$
(b) $f(z) = 3x + y + i(3y - x)$

4. Determine the singular points (singularites) of the function below and state why the function is analytic everywhere except at those points.
(a) $f(z) = \dfrac{z^3 + i}{z^2 - 3z + 2}$
(b) $f(z) = \dfrac{z^2 + 1}{(z+2)(z^2 + 2z + 2)}$

5. Show that u(x,y) is harmonic in some domain and find a harmonic conjugate v(x,y) when
(a) $u(x,y) = 2x - x^3 + 3xy^2$
(b) $u(x,y) = \sinh x \sin y$

6. Suppose that v is a harmonic conjugate of u in a domain and also that u is a harmonic conjugate of v in D. Show that both u and v must be constant in D.

7. Can the function $u(x,y) = x^2 + y$ be the real part of an analytic function? Justify your answer.