Math 408(42-49) Homework 5 Name & Number:

1. Evaluate the following integrals.

(a) $\int_0^1 (1+it)^2 dt$

(b) $\int_1^2 (\frac{1}{t} - i)^2 dt$

(c) $\int_0^\infty e^{-zt} dt$, $\operatorname{Re} z > 0$

2. Show that if C is the boundary of the triangle with vertices at the points 0,3i, and -4, oriented in the counterclockwise direction, then $|\oint_C (e^z - \bar{z})dz| \le 60$.

3. Let C_R be the circle $|z| = R$ (R>1), described in the counterclockwise direction. Show that

$|\oint_C \frac{\operatorname{Log} z}{z^2} dz| < 2\pi(\frac{\pi + \ln R}{R})$, and then use L'hopital rule to show that the value of this integral tends to zero as R approach to infinity.

4. Evaluate the following contour integral. If you used the theorem mentioned in class, please state it clearly.

(a) $\int_i^{i/2} e^{\pi z} dz$

(b) $\oint_C z^n dz$, C is a contour from a point z_1 to a point z_2.

(c) $\oint_C f(z)dz$, where $f(z)$ is the principal branch of the power function z^i, and C is the semicircle $z = e^{i\theta} (0 \le \theta \le \pi)$.

5. Complete the proof of the theorem discussed in class from the page 146.