Math 408(50-56) Homework 6 Name & Number:

1. If P(z) is a polynomial of degree n, prove that $\oint_{|z|=2} \frac{P(z)}{(z-1)^{n+2}} dz = 0$.

2. Evaluate the following integral. If you used the theorem mentioned in class, please state it clearly.

(a) $\oint_C \frac{z^3}{z-3} dz$, where C is the unit disk |z|=1.

(b) $\oint_{|z|=3} \frac{e^z \sin z}{(z-2)^2} dz$

(c) $\oint_{|z|=3} \frac{z^3+3z-1}{(z-1)(z+2)} dz$

3. Explain why the following is true.

$\oint_{C_1} \frac{z+2}{\sin(z/2)} dz = \oint_{C_2} \frac{z+2}{\sin(z/2)} dz$ where C_1 is the positively oriented boundary of the square whose sides lie along the lines $x = \pm 1,\ y = \pm 1$ and C_2 is the positively oriented circle $|z| = 4$.

4. Let C denote the positively oriented boundary of the square whose sides lie along the lines $x = \pm 2,\ y = \pm 2$. Evaluate each of these integrals.

(a) $\oint_C \frac{e^{-z} dz}{z - (\pi i/2)}$

(b) $\oint_C \frac{z dz}{2z+1}$

(c) $\oint_C \frac{\tan(z/2) dz}{(z-x_0)^2}$ where $-2 < x_0 < 2$.

5. Show that if f is analytic within and on a simple closed contour C and z_0 is not on C, then

$$\int_C \frac{f'(z) dz}{z-z_0} = \int_C \frac{f(z) dz}{(z-z_0)^2}$$

6. Let C be the circle $|z| = 3$, counterclockwise oriented. For $g(z) = \int_C \frac{2s^2 - s - 2}{s-z} ds,\ \ |z| \neq 3,$

(a) Find the value of $g(2)$.

(b) What is the value of $g(z)$ if z is outside the circle $|z| = 3$?