Math 408(57-64)   Homework 7   Name & Number:

1. Find the Taylor series expansion of the following and also give the region of the representation. It is Ok to use the known series for the functions e^z, $\frac{1}{1-z}$ $(|z|<1)$, $\sin z$, $\cos z$.

(a) $f(z)=e^{-z}$ at 0

(b) $g(z)=\frac{z}{z^4+9}$ at 0.

(c) $h(z)=\frac{z+1}{z-1}$ at -1.

(d) $k(z)=\frac{1}{z}$ at i.

(e) $\sin(z^2)$ at 0.

2. Prove: Taylor theorem in pages 187-189

3. Suppose that $f(z)$ is an entire function and that $|f(z)|\le Mr^{\lambda}$ $(|z|=r)$ for some nonnegative real number λ. Then $f(z)$ is a polynomial of degree at most λ.

4. Find an example of real variable function that fails the Liouville's theorem.

5. Suppose a, b, and c are distinct complex numbers. Fid a Taylor series expansion for $f(z)=\frac{1}{(z-a)(z-b)}$ about the point $z=c$, and determine its radius of convergence.

6. Prove: If $\sum_{n=1}^{\infty}|z_n|$ converges, then $\sum_{n=1}^{\infty} z_n$ converges.